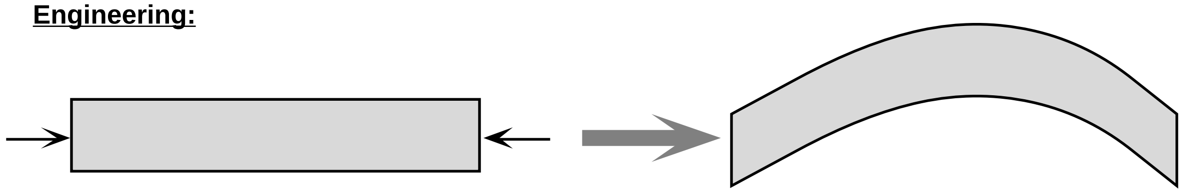
Lecture 1—Introduction, Scale, & Basic Terminology

1.1 Introduction

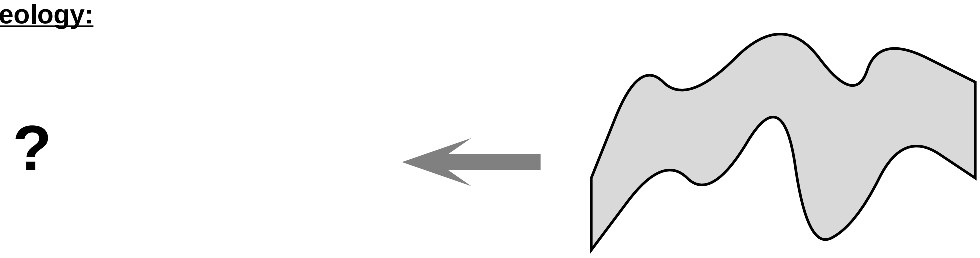
Structural Geology is the study of deformed rocks. To do so, we define the geometries of rock bodies in three dimensions. Then, we measure or infer the translations, rotations, and strains experienced by rocks both during, and particularly since, their formation based on indicators of what they looked like prior to their deformation. Finally, we try to infer the stresses that produced the deformation based on our knowledge of material properties. Structure is closely related to various fields of engineering mechanics, structural engineering, and material science.

But, there is a big difference: In structural geology, we deal almost exclusively with the end product of deformation in extremely heterogeneous materials. Given this end product, we try to *infer* the processes by which the deformation occurred. In engineering, one is generally more interested in the effect that various, known or predicted, stress systems will produce on undeformed, relatively homogeneous materials.

Engineering:



Structural Geology:



Key Point: What we study in structural geology is strain and its related translations and rotations; this is the end product of deformation. We never observe stress directly or the forces responsible for the deformation. A famous structural geologist, John Ramsay, once said that "as a geologist, I don't believe in stress". This view is perhaps too extreme -- stress certainly does exist, but we cannot measure it directly. Stress is an instantaneous entity; it exists only in the moment that it is applied. In Structural Geology we study geological materials that were deformed in the past, whether it be a landslide that formed two hours ago or a fold that formed 500 Ma ago. *The stresses that were responsible for that deformation are no* *longer present*. Even when the stresses of interest are still present, such as in the test of the strength of a concrete block in an engineering experiment, you cannot measure stress directly. What you do is measure the strain of some material whose material response to stress, or rheology, is very well known.

If you learn nothing else in this course, it should be the distinction between stress and strain, and what terms are appropriate to each:

Stress	Strain
compression	shortening (contraction)
tension	lengthening (extension)

note that terms in the same row are **not** equal but have somewhat parallel meanings. As we will see later in the course, the relations among these terms is quite

1.2 Levels of Structural Study

There are three basic level at which one can pursue structural geology and these are reflected in the organization of this course:

- **Geometry** basically means how big or extensive something is (size or magnitude) and/or how its dimensions are aligned in space (orientation). We will spend only a little time during lecture on the geometric description of structures because most of the lab part of this course is devoted to this topic.

- **Kinematics** is the description of movements that particles of material have experienced during their history. Thus we are comparing two different states of the material, whether they be the starting point and ending point or just two intermediate points along the way.

- **Mechanics** implies an understanding of how forces applied to a material have produced the movements of the particles that make up the material.

1.3 Types of Structural Study

- Observation of natural structures, or deformed features in rock. This observation can take place at many different scales, from the submicroscopic to the global. Observation usually involves the description of the geometry and orientations of individual structures and their relations to other structures. Also generally involves establishing of the timing relations of structures (i.e. their order of formation, or the time it took for one feature to form).

- Experimental -- an attempt to reproduce under controlled laboratory conditions various features similar to those in naturally deformed rocks. The aim of experimental work is to gain insight into the stress systems and processes that produced the deformation. Two major drawbacks: (1) in the real earth, we seldom know all of the possible factors effecting the deformation (P, T, t, fluids, etc.); (2) More important, real earth processes occur at rates which are far slower than one can possibly reproduce in the laboratory (Natural rates: 10^{-12} to 10^{-18} sec^{-1}; in lab, the slowest rates: 10^{-6} - 10^{-8} sec^{-1})

- Theoretical -- application of various physical laws of mechanics and thermodynamics, through analytical or numerical methods, to relatively simple structural models. The objective of this modeling is to duplicate, theoretically, the geometries or strain distributions of various natural features. Main problem is the complexity of natural systems.

1.4 Importance of Scale

1.4.1 Scale Terms

Structural geologists view the deformed earth at a variety of different scales. Thus a number of general terms are used to refer to the different scales. All are vague in detail. Importantly, *all depend on the vantage point of the viewer*:

- Global -- scale of the entire world. ~10^4-10^5 km (circumference = 4 x 10^4 km)

- Regional or Provincial -- poorly defined; generally corresponds to a physiographic province (e.g. the Basin and Range) or a mountain belt 10^3-10^4 km (e.g. the Appalachians).

- Macroscopic or Map Scale -- Bigger than an area you can see standing in one place on the ground. 10^0-10^2 km (e.g. the scale of a 7.5' quadrangle map)

- Mesoscopic -- features observable in one place on the ground. An outcrop of hand sample scale. 10^{-5}-10^{-1} km (1 cm - 100s m) (e.g. scale of a hand sample)

- Microscopic -- visible with an optical microscope. 10^{-8}-10^{-6} km

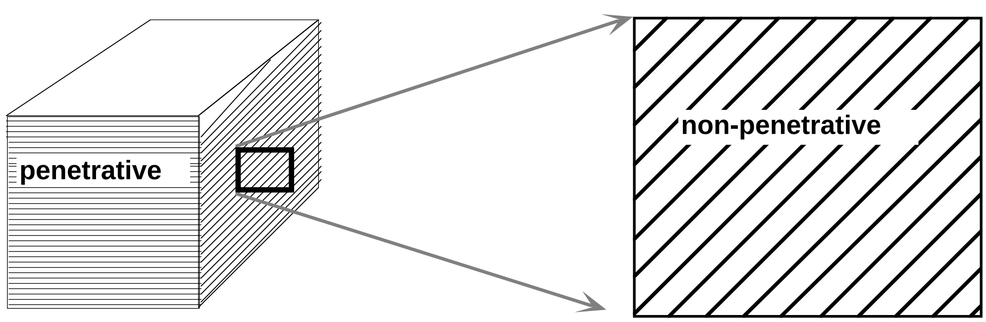
- Submicroscopic -- not resolvable with a microscope but with TEM, SEM etc. < 10^{-8} km.

Two additional terms describe how pervasive a feature or structure is at the scale of observation:

- Penetrative -- characterizes the entire body of rock at the scale of observation

- Non-penetrative -- Does not characterize the entire body of rock

These terms are totally scale dependent. A cleavage can be penetrative at one scale (i.e. the rock appears to be composed of nothing but cleavage planes), but non-penetrative at another (e.g. at a higher magnification where one sees coherent rock between the cleavage planes):



The importance of scale applies not only to description, but also to our mechanical analysis of structures. For example, it may not be appropriate to model a rock with fractures and irregularities at the mesoscopic scale as an elastic plate, whereas it may be totally appropriate at a regional scale. There are no firm rules about what scale is appropriate for which analysis.

1.4.2 Scale Invariance, Fractals

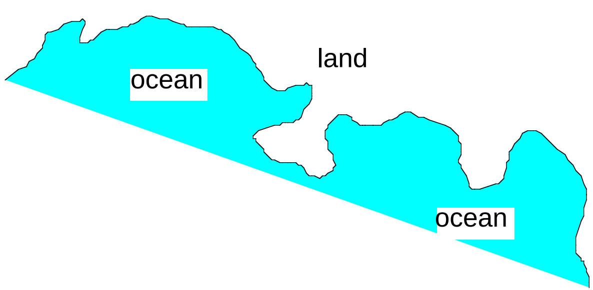
Many structures occur over a wide range of scales. Faults, for example, can be millimeters long or they can be 1000s of kilometers long (and all scales in between). Likewise, folds can be seen in thin sections under the microscope or they can be observed at map scale, covering 100s of square kilometers.

Geologists commonly put a recognizable feature such as a rock hammer or pencil in a photograph ("rock hammer for scale") because otherwise, the viewer might not know if s/he was looking at a 10 cm high outcrop or a 2000 m high cliff. Geologic maps commonly show about the same density of faults, regardless of whether the map has a scale of 1:5,000,000 or 1:5,000.

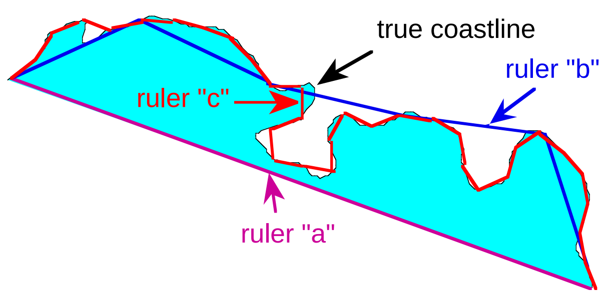
These are all examples of the <u>scale invariance</u> of certain structures. Commonly, there is a consistent relationship between the size of something and the frequency with which it occurs or the size of the measuring stick that you use to measure it with. The exponent in this relationship is called the <u>fractal dimension</u>.

The term, fractal, was first proposed by B. Mandelbrot (1967). He posed a very simple question: "How long is the coast of Britain?" Surprisingly, at first, there is no answer to this question; the coast of Britain has an undefinable length. *<u>The length of the coast of Britain depends on the scale at which you measure it</u>*. The longer the measuring stick, the shorter the length as illustrated by the picture below. On a globe with a scale of 1:25,000,000, the shortest distance you can effectively measure (i.e. the measuring stick) is 10s of kilometers long. Therefore at that scale you cannot measure all of the little bays and promontories. But with accurate topographic maps at a scale of 1:25,000, your measuring stick can be as small as a few tens of meters and you can include much more detail than previously. Thus, your measurement of the coast will be longer. You can easily imagine extending this concept down to the scale of a single grain of sand, in which case your measured length would be immense!

<u>"True" Geography</u>



<u>Measurements with Successively Smaller Rulers</u>



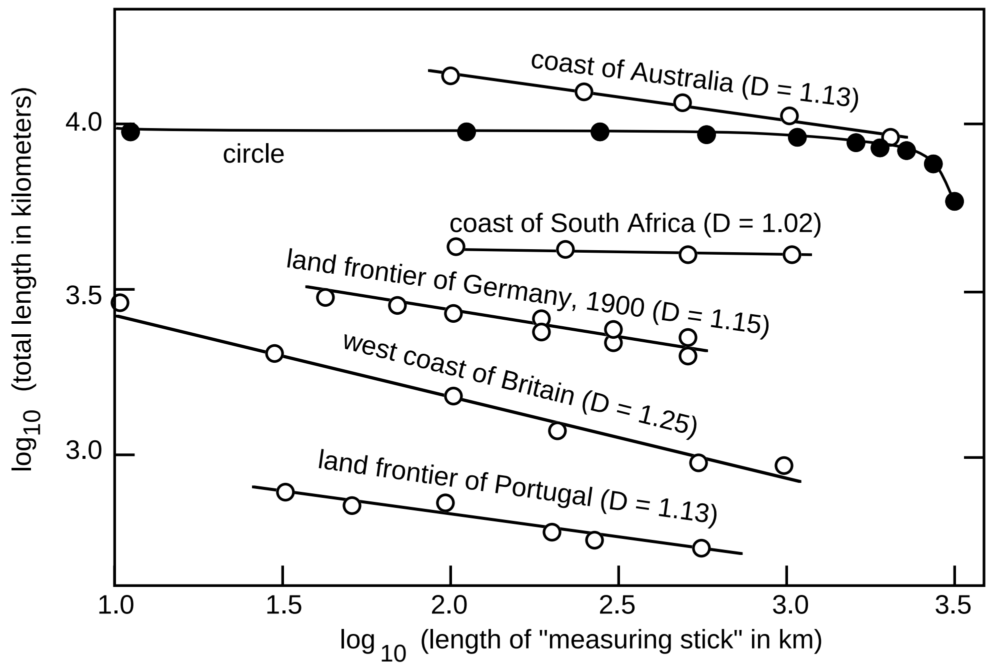
<u>Length of coastline as determined with:</u>

Mandelbrot defined the fractal dimension, D, according to the following equation:

$$L(G) \sim G^{1-D}$$

where G is the length of the measuring stick and L(G) is the length of the coastline that you get using that measuring stick.

The plot below, from Mandelbrot's original article, shows this scale dependence for a number of different coasts in log-log form.



Fractals have a broad range of applications in structural geology and geophysics. The relation between earthquake frequency and magnitude, m, is a log linear relation:

$$\log N = -b\,m + a$$

where N is the number of earthquakes in a given time interval with a magnitude m or larger. Empirically, the value of b (or "b-value") is about 1, which means that, for every magnitude 8 earthquakes, there are 10 magnitude 7 earthquakes; for every magnitude 7 there are 10 magnitude 6; etc. The strain released during an earthquake is directly related to the moment of the earthquake, and moment, M, and magnitude are related by the following equation:

$$\log M = c\,m + d$$

where c and d are constants. Thus, the relation between strain release and number is log-log or fractal:

$$\log N = \frac{-b}{c}\log M + \left(a + \frac{bd}{c}\right)$$

LECTURE 2 —COORDINATE SYSTEMS, ETC.

2.1 Introduction

As you will see in lab, structural geologists spend a lot of time describing the orientation and direction of structural features. For example, we will see how to describe the strike and dip of bedding, the orientation of a fold axis, or how one side of a fault block is displaced with respect to the other. As you might guess, there are several different ways to do this:

- **plane trigonometry**.
- **spherical trigonometry**
- **vector algebra**

All three implicitly require a coordinate system. Plane trigonometry works very well for simple problems but is more cumbersome, or more likely impossible, for more complex problems. Spherical trigonometry is much more flexible and is the basis for a wonderful graphical device which all structural geologists come to love, the stereonet. In lab, we will concentrate on both of these methods of solving structural problems.

The third method, vector algebra, is less familiar to many geologist and is seldom taught in introductory courses. But it is so useful, and mathematically simple, that I wanted to give you an introduction to it. Before that we have to put the term, **vector**, in some physical context, and talk about coordinate systems.

2.2 Three types of physical entities

Let's say we measure a physical property of something: for example, the density of a rock. Mathematically, what is the number that results? Just a single number. It doesn't matter where the sample is located or how it is oriented, it is still just a single number. Quantities like these are called **scalars**.

Some physical entities are more complex because they do depend on their position in space or their *orientation with respect to some coordinate system*. For example, it doesn't make much sense to talk about displacement if your don't know where something was originally and where it ended up after the displacement. Quantities like these, where the direction is important, are called **vectors**.

Finally, there are much more complex entities, still, which also must be related to a coordinate system. These are "fields" of vectors, or things which vary in all different directions. These are called **tensors**.

	Examples
Scalars	mass, volume, density, temperature
Vectors	velocity, displacement, force, acceleration, poles to planes, azimuths
Tensors	stress, strain, thermal conductivity, magnetic susceptibility

Most of the things we are interested in Structural Geology are vectors or tensors. And that means that we have to be concerned with coordinate systems and how they work.

2.3 Coordinate Systems

Virtually everything we do in structural geology explicitly or implicitly involves a coordinate system.

- When we plot data on a map each point has a latitude, longitude, and elevation. Strike and dip of bedding are given in azimuth or quadrant with respect to north, south, east, and west and with respect to the horizontal surface of the Earth approximated by sea level.

- In the western United States, samples may be located with respect to township and range.

- More informal coordinate systems are used as well, particularly in the field. The location of an observation or a sample may be described as "1.2 km from the northwest corner fence post and 3.5 km from the peak with an elevation of 6780 m at an elevation of 4890 m."

A key aspect, but one which is commonly taken for granted, of all of these ways of reporting a location is that *they are interchangeable*. The sample that comes from near the fence post and the peak could just as easily be described by its latitude, longitude, and elevation or by its township, range and elevation. Just because I change the way of reporting my coordinates (i.e. change my coordinate system) does not mean that the physical location of the point in space has changed.

2.3.1 Spherical versus Cartesian Coordinate Systems

Because the Earth is nearly spherical, it is most convenient for structural geologists to record their observations in terms of **spherical coordinates**. Spherical coordinates are those which are referenced to a sphere (i.e. the Earth) and are fixed by two angles and a distance, or radius (Fig. 2.1). In this case the two angles are latitude, ϕ, and longitude, θ, and the radius is the distance, r, from the center of the Earth (or in elevation which is a function of the distance from the center). The rotation axis is taken as one axis (from which the angle ϕ or its complement is measured) with the other axis at the equator and arbitrarily coinciding with the line of longitude which passes through Greenwich, England. The angle θ is measured from this second axis.

We report the azimuth as a function of angle from north and the inclination as the angle between a tangent to the surface and the feature of interest in a vertical plane. A geologist can make these orientation measurements with nothing more than a simple compass and clinometer because the Earth's magnetic poles are close to its rotation axis and therefore close to one of the principal axes of our spherical coordinate system.

Although a spherical coordinate system is the easiest to use for collecting data in the field, it is not the simplest for accomplishing a variety of calculations that we need to perform. Far simpler, both conceptually and computationally, are **rectangular Cartesian coordinates**. This coordinate system is composed of three mutually perpendicular axes. Normally, one thinks of plotting a point by its distance from the three axes of the Cartesian coordinate system. As we shall see below, a feature can equally well be plotted by the angles that a vector, connecting it to the origin, makes with the axes. If we can assume that the portion of the Earth we are studying is sufficiently small so that our horizontal reference surface is essentially perpendicular to the radius of the Earth, then we can solve many different problems in structural geology simply and easily by expressing them in terms of Cartesian, rather than spherical, coordinates. Before we can do this however, there is an additional aspect of coordinate systems which we must examine.

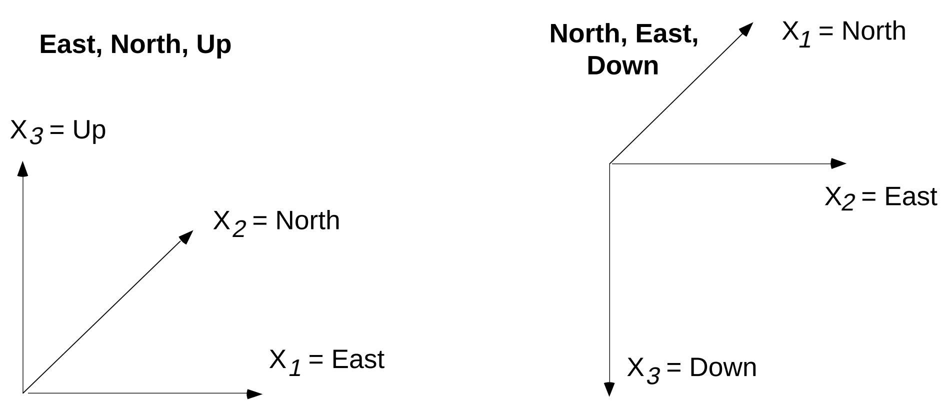
2.3.2 Right-handed and Left-handed Coordinate Systems

The way that the axes of coordinate systems are *labelled* is not arbitrary. In the case of the Earth, it matters whether we consider a point which is below sea level to be positive or negative. That's crazy, you say, everybody knows that elevations above sea level are positive! If that were the case, then why do structural geologists commonly measure positive angles *downward* from the horizontal? Why is it that mineralogists use an upper hemisphere stereographic projection whereas structural geologists use the lower hemisphere? The point is that it does not matter which is chosen so long as one is clear and consistent. There are some simple conventions in the labeling of coordinate axes which insure that consistency.

Basically, coordinate systems can be of two types. **Right-handed** coordinates are those in which, if you hold your hand with the thumb pointed from the origin in the positive direction of the first axis, your fingers will curl from the positive direction of the second axis towards the positive direction of the third axis (Fig. 2.2). A **left-handed** coordinate system would function the same except that the left hand is used. To make the coordinate system in Fig. 2.2 left handed, simply reverse the positions of the X_2 and X_3 axes. By convention, the preferred coordinate system is a right-handed one and that is the one we shall use.

2.3.3 Cartesian Coordinate Systems in Geology

What Cartesian coordinate systems are appropriate to geology? Sticking with the right-handed convention, there are two obvious choices, the primary difference being whether one regards up or down as positive:



Cartesian coordinates commonly used in geology and geophysics

In general, the north-east-down convention is more common in structural geology where positive angles are measured downwards from the horizontal. In geophysics, the east-north-up convention is more customary. Note that these are not the only possible right-handed coordinate systems. For example, west-south-up is also a perfectly good right-handed system although it, and all the other possible combinations are seldom used.

2.4 Vectors

Vectors form the basis for virtually all structural calculations so it's important to develop a very clear, intuitive feel for them. Vectors are a physical quantity that have a magnitude and a direction; they can be defined only with respect to a given coordinate system.

2.4.1 Vectors vs. Axes

At this point, we have to make a distinction between vectors, which are lines with a direction (i.e. an arrow at one end of the line) and **axes**, which are lines with no directional significance. For example, think about the lineation that is made by the intersection between cleavage and bedding. That line, or axis, certainly has a specific orientation in space and is described with respect to a coordinate system, but there is no difference between one end of the line and the other.[1] The hinge — or axis — of a cylindrical fold is another example of a line which has no directional significance. Some common geological examples of vectors which *cannot* be treated as axes, are the slip on a fault (i.e. displacement of piercing points), paleocurrent indicators (flute cast, etc.), and paleomagnetic poles.

2.4.2 Basic Properties of Vectors

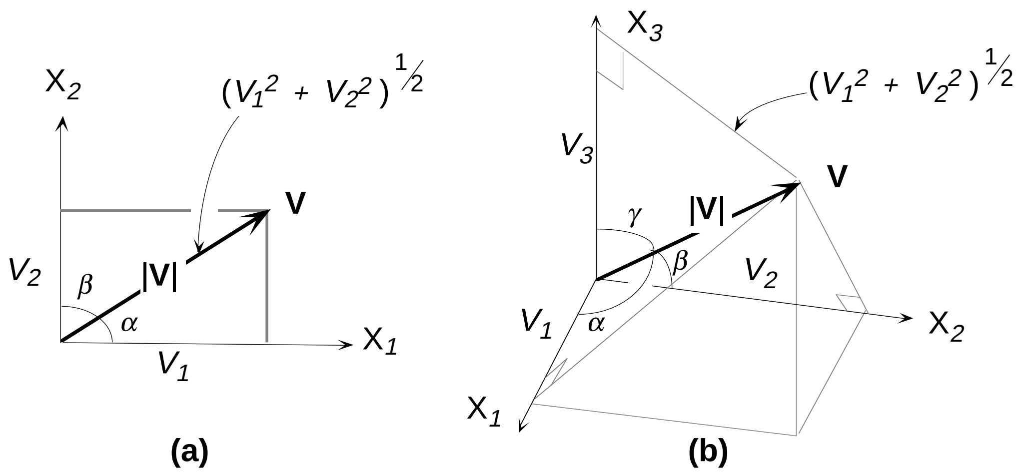
Notation. Clearly, with two different types of quantities — scalars and vectors — around, we need a shorthand way to distinguish between them in equations. Vectors are generally indicated by a letter with a bar, or in these notes, in bold face print (which is sometimes known as **symbolic** or **Gibbs notation**):

$$\bar{V} = \mathbf{V} = [V_1, V_2, V_3] \qquad \text{(eqn. 1)}$$

[1][It should be noted that, when structural geologists use a lower hemisphere stereographic projection exclusively we are automatically treating all lines as axes. To plot lines on the lower hemisphere, we arbitrarily assume that all lines point downwards. Generally this is not an issue, but consider the problem of a series of complex rotations involving paleocurrent directions. At some point during this process, the current direction may point into the air (i.e. the upper hemisphere). If we force that line to point into the lower hemisphere, we have just reversed the direction in which the current flowed! Generally poles to bedding are treated as axes as, for example, when we make a π-diagram. This, however, is not strictly correct. There are really two bedding poles, the vector which points in the direction of stratal younging and the vector which points towards older rocks.]

Vectors in three dimensional space have three components, indicated above as V_1, V_2, and V_3. These components are scalars and, in a Cartesian Coordinate system, they give the magnitude of the vector in the direction of, or projected onto each of the three axes (b). Because it is tedious to write out the three components all the time a shorthand notation, known as **indicial notation**, is commonly used:

$$V_i \text{ , where } [i = 1, 2, 3]$$



Components of a vector in Cartesian coordinates (a) in two dimensions and (b) in three dimensions

<u>*Magnitude of a Vector*</u>. The magnitude of a vector is, graphically, just the length of the arrow. It is a scalar quantity. In two dimensions it is quite easy to see that the magnitude of vector **V** can be calculated from the Pythagorean Theorem (the square of the hypotenuse is equal to the sum of the squares of the other two sides). This is easily generalized to three dimensions, yielding the general equation for the magnitude of a vector:

$$V = |\mathbf{V}| = (V_1^2 + V_2^2 + V_3^2)^{1/2} \qquad \text{(eqn. 2)}$$

<u>*Unit Vector*</u>. A unit vector is just a vector with a magnitude of one and is indicated by a triangular hat: $\widehat{\mathbf{V}}$. Any vector can be converted into a unit vector parallel to itself by dividing the vector (and its components) by its own magnitude.

$$\hat{\mathbf{V}} = \left[\frac{V_1}{V}, \frac{V_2}{V}, \frac{V_3}{V}\right] \qquad \text{(eqn. 3)}$$

Direction Cosines. The cosine of the angle that a vector makes with a particular axis is just equal to the component of the vector along that axis divided by the magnitude of the vector. Thus we get

$$\cos\alpha = \frac{V_1}{V}, \quad \cos\beta = \frac{V_2}{V}, \quad \cos\gamma = \frac{V_3}{V} \quad . \qquad \text{(eqn. 4)}$$

Substituting equation eqn. 4 into equation eqn. 3 we see that a unit vector can be expressed in terms of the cosines of the angles that it makes with the axes. These cosines are known as **direction cosines**:

$$\hat{\mathbf{V}} = \left[\cos\alpha,\ \cos\beta,\ \cos\gamma\right]. \qquad \text{(eqn. 5)}$$

Direction Cosines and Structural Geology. The concept of a unit vector is particularly important in structural geology where we so often deal with orientations, but not sizes, of planes and lines. Any orientation can be expressed as a unit vector, whose components are the direction cosines. For example, in a north-east-down coordinate system, a line which has a 30° plunge due east (090°, 30°) would have the following components:

$\cos\alpha = \cos 90° = 0.0$ [α is the angle with respect to north]

$\cos\beta = \cos 30° = 0.866$ [β is the angle with respect to east]

$\cos\gamma = (\cos 90° - 30°) = 0.5$ [γ is the angle with respect to down]

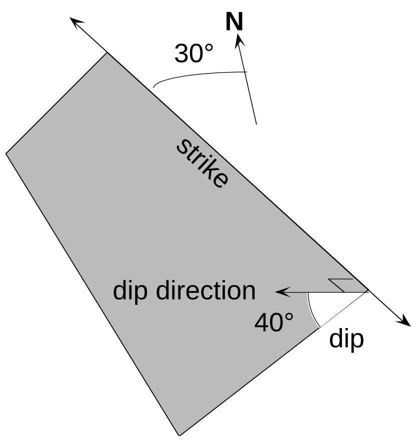
or simply $[\cos\alpha, \cos\beta, \cos\gamma] = [\,0.0\,,\ 0.866\,,\ 0.5\,]$.

For the third direction cosine, recall that the angle is measured with respect to the vertical, whereas plunge is given with respect to the horizontal.

2.4.3 Geologic Features as Vectors

Virtually all structural features can be reduced to two simple geometric objects: lines and planes. Lines can be treated as vectors. Likewise, because there is only one line which is perpendicular to a plane, planes — or more strictly, poles to planes — can also be treated as vectors. The question now is, how do we convert from orientations measured in spherical coordinates to Cartesian coordinates?

Data Formats in Spherical Coordinates. Before that question can be answered, however, we have to examine for a minute how orientations are generally specified in spherical coordinates (Fig. 2.6). In North America, planes are commonly recorded according to their strike and dip. But, the strike can correspond to either of two directions 180° apart, and dip direction must be fixed by specifying a geographic quadrant. This can lead to ambiguity which, if we are trying to be quantitative, is dangerous. There are two methods of recording the orientation of a plane that avoids this ambiguity. First, one can record the strike azimuth such that the dip direction is always clockwise from it, a convention known as the **right-hand rule**. This tends to be the convention of choice in North America because it is easy to determine using a Brunton compass. A second method is to record the **dip and dip direction**, which is more common in Europe where compasses make this measurement directly. Of course, the pole also uniquely defines the plane, but it cannot be measured directly off of either type of compass.



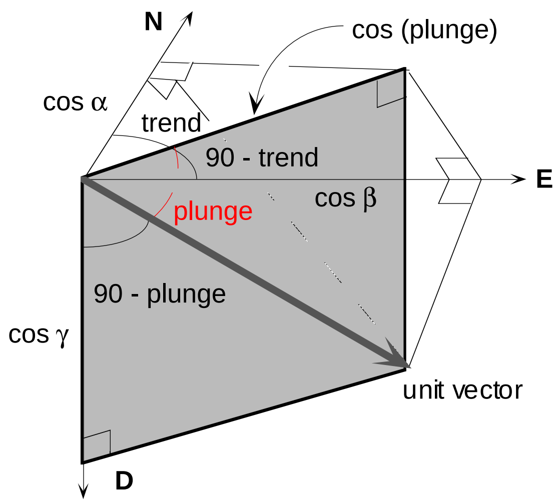
Quadrant:	N 30 W, 40 SW
Azimuth & dip quadrant:	330, 40 SW
Azimuth, right-hand rule:	**150, 40**
Dip azimuth & dip:	**240, 40**
Pole trend & plunge:	**060, 50**

Alternative ways of recording the strike and dip of a plane. The methods which are not subject to potential ambiguity are shown in bold face type.

Lines are generally recorded in one of two ways. Those associated with planes are commonly recorded by their orientation with respect to the strike of the plane, that is, their pitch or rake. Although this way is commonly the most convenient in the field, it can lead to considerable ambiguity if one is not careful because of the ambiguity in strike, mentioned above, and the fact that pitch can be either of two complementary angles. The second method — recording the trend and plunge directly — is completely unambiguous as long as the lower hemisphere is always treated as positive. Vectors which point into the upper hemisphere (e.g. paleomagnetic poles) can simply be given a negative plunge.

Conversion from Spherical to Cartesian Coordinates. The relations between spherical and Cartesian coordinates are shown in Fig. 2.7. Notice that the three angles α, β, and γ are measured along great circles between the point (which represents the vector) and the *positive* direction of the axis of the Cartesian coordinate system. Clearly, the angle γ is just equal to 90° minus the plunge of the line. Therefore ,

$$\cos\gamma = \cos(90 - \text{plunge}) = \sin(\text{plunge}) \qquad \text{(eqn. 6a)}$$



Perspective diagram showing the relations between the trend and plunge angles and the direction cosines of the vector in the Cartesian coordinate system. Gray plane is the vertical plane in which the plunge is measured.

The relations between the trend and plunge and the other two angles are slightly more difficult to calculate. Recall that we are dealing just with orientations and therefor the vector of interest (the bold arrowhead in Fig. 2.8) is a *unit vector*. Therefore, from simple trigonometry the horizontal line which corresponds to the trend azimuth is equal to the cosine of the plunge. From here, it is just a matter of solving for the horizontal triangles in Fig. 2.8:

$$\cos\alpha = \cos(\text{trend})\cos(\text{plunge}), \qquad \text{(eqn. 6b)}$$

$$\cos\beta = \cos(90 - \text{trend})\cos(\text{plunge}) = \sin(\text{trend})\cos(\text{plunge}). \qquad \text{(eqn. 6c)}$$

These relations, along with those for poles to planes, are summarized in Table 1:

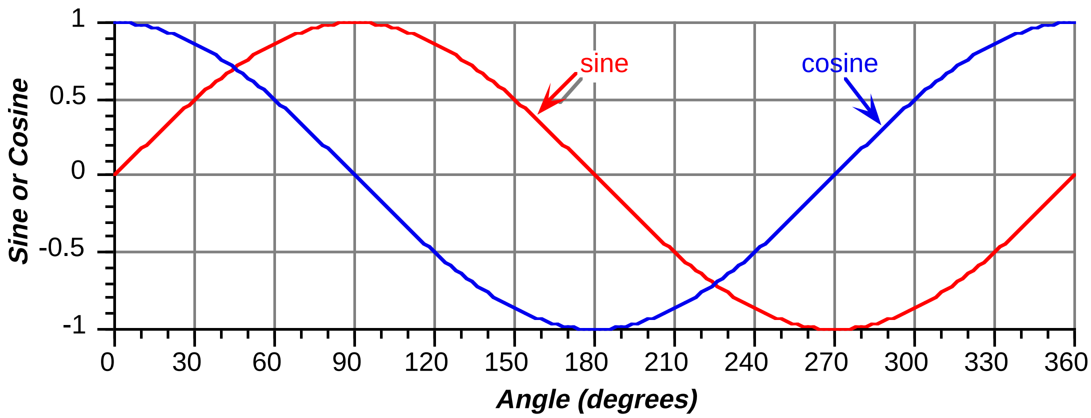
Table 1: Conversion from Spherical to Cartesian Coordinates

Axis	Direction Cosine	Lines	Poles to Planes (right-hand rule)
North	cos α	cos(trend)*cos(plunge)	sin(strike)*sin(dip)
East	cos β	sin(trend)*cos(plunge)	–cos(strike)*sin(dip)
Down	cos γ	sin(plunge)	cos(dip)

The signs of the direction cosines vary with the quadrant. Although it is not easy to see an orientation expressed in direction cosines and immediately have an intuitive feel how it is oriented in space, one can quickly tell what quadrant the line dips in by the signs of the components of the vector.

For example, the vector, [–0.4619, –0.7112, 0.5299], represents a line which plunges into the southwest quadrant (237°, 32°) because both cos α and cos β are negative.

Understanding how the signs work is very important for another reason. Because it *is* difficult to get an intuitive feel for orientations in direction cosine form, after we do our calculations we will want to convert from Cartesian *back* to spherical coordinates. This can be tricky because, for each direction cosine, there will be two possible angles (due to the azimuthal range of 0 - 360°). For example, if cos α = –0.5736, then α = 125° *or* α = 235°. In order to tell which of the two is correct, one must look at the value of cos β; if it is negative then α = 235°, if positive then α = 125°. When you use a calculator or a computer to calculate the inverse cosine, it will only give you one of the two possible angles (generally the smaller of the two). You must determine what the other one is knowing the cyclicity of the sine and cosine functions.



Graph of sine and cosine functions for 0 - 360°. The plot emphasizes that for every positive (or negative) cosine, there are two possible angles.

2.4.4 Simple Vector Operations

Scalar Multiplication. To multiply a scalar times a vector, just multiply each component of the vector times the scalar.

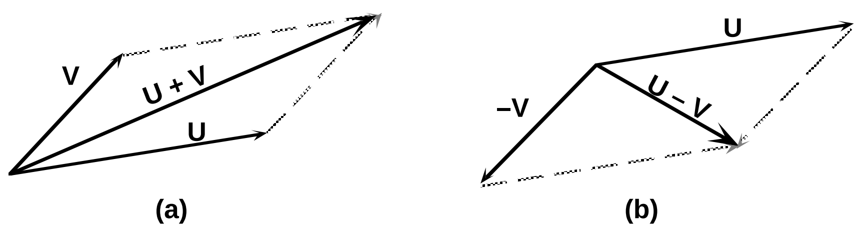
$$xV = [\, xV_1, xV_2, xV_3 \,] \qquad \text{(eqn. 7)}$$

The most obvious application of scalar multiplication in structural geology is when you want to reverse the direction of the vector. For example, to change the vector from upper hemisphere to lower (or vice versa) just multiply the vector (i.e. its components) by –1. The resulting vector will be parallel to the original and will have the same length, but will point in the opposite direction.

Vector Addition. To add two vectors together, you sum their components:

$$U + V = V + U = [\, V_1 + U_1 ,\ V_2 + U_2 ,\ V_3 + U_3 \,] . \qquad \text{(eqn. 8)}$$

Graphically, vector addition obeys the parallelogram law whereby the resulting vector can be constructed by placing the two vectors to be added end-to-end:



(a) Vector addition and (b) subtraction using the parallelogram law.

Notice that the order in which you add the two vectors together makes no difference. Vector subtraction is the same as adding the negative of one vector to the positive of the other.

2.4.5 Dot Product and Cross Product

Vector algebra is remarkably simple, in part by virtue of the ease with which one can *visualize* various operations. There are two operations which are unique to vectors and which are of great importance in structural geology. If one understands these two, one has mastered the concept of vectors. They are the **dot product** and the **cross product**.

Dot Product. The dot product is also called the "**scalar product**" because this operation produces a scalar quantity. When we calculate the dot product of two vectors the result is the magnitude of the first vector times the magnitude of the second vector times the cosine of the angle between the two:

$$\mathbf{U} \bullet \mathbf{V} = \mathbf{V} \bullet \mathbf{U} = U\,V \cos\theta = U_1V_1 + U_2V_2 + U_3V_3 , \qquad \text{(eqn. 9)}$$

The physical meaning of the dot product is the length of **V** times the length of **U** as projected onto **V** (that is, the length of **U** in the direction of **V**). Note that the dot product is zero when **U** and **V** are perpendicular (because in that case the length of **U** projected onto **V** is zero). The dot product of a vector with itself is just equal to the length of the vector:

$$\mathbf{V} \cdot \mathbf{V} = V = |\mathbf{V}|. \qquad \text{(eqn. 10)}$$

Equation (eqn. 9) can be rearranged to solve for the angle between two vectors:

$$\theta = \cos^{-1}\left(\frac{\mathbf{U} \bullet \mathbf{V}}{UV}\right). \qquad \text{(eqn. 11)}$$

This last equation is particularly useful in structural geology. As stated previously, all orientations are treated as unit vectors. Thus when we want to find the angle between any two lines, the product of the two magnitudes, UV, in equations (eqn. 9) and (eqn. 11) is equal to one. Upon rearranging equations (eqn. 11), this provides a simple and extremely useful equation for calculating the angle between two lines:

$$\theta = \cos^{-1}(\cos\alpha_1 \cos\alpha_2 + \cos\beta_1 \cos\beta_2 + \cos\gamma_1 \cos\gamma_2). \qquad \text{(eqn. 12)}$$

<u>Cross Product</u>. The result of the cross product of two vectors is another vector. For that reason, you will often see the cross product called the "**vector product**". The cross product is conceptually a little more difficult than the dot product, but is equally useful in structural geology. It's primary use is when you want to calculate the orientation of a vector that is perpendicular to two other vectors. The resulting perpendicular vector is parallel to the unit vector, $\hat{\mathbf{l}}$, and has a magnitude equal to the product of the magnitude of each vector times the sine of the angle between them. The new vector obeys a right-hand rule with respect to the other two.

$$V \times U = V \wedge U = (V\, U \sin\theta)\, \hat{\mathbf{l}} \qquad \text{(eqn. 13)}$$

and

$$V \times U = [V_2U_3 - V_3U_2,\ V_3U_1 - V_1U_3,\ V_1U_2 - V_2U_1] \qquad \text{(eqn. 14)}$$

The cross product is best illustrated with a diagram, which relates to the above equations:

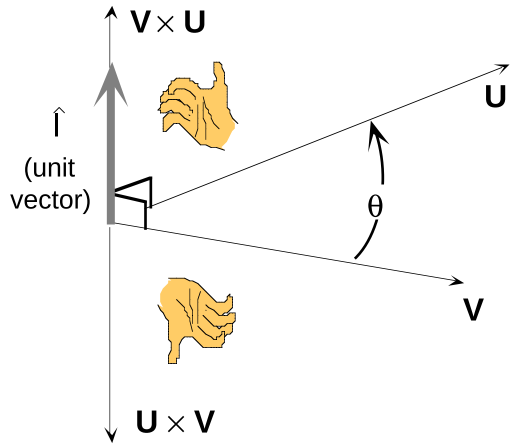


Diagram illustrating the meaning of the cross-product. The hand indicates the right-hand rule convention; for $\mathbf{V} \times \mathbf{U}$, the finger curl from $\mathbf{V}$ towards $\mathbf{U}$ and the thumb points in the direction of the resulting vector, which is parallel to the unit vector $\hat{\mathbf{I}}$. Note that $\mathbf{V} \times \mathbf{U} = -\mathbf{U} \times \mathbf{V}$

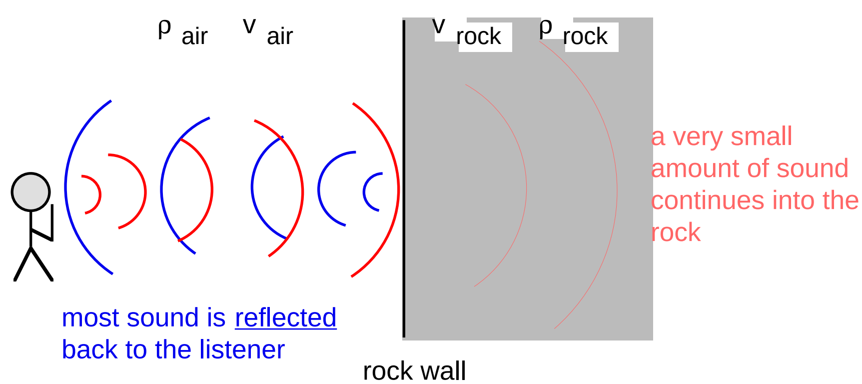
Lecture 3 — Descriptive Geometry: Seismic Reflection

3.1 Echo Sounding

Geology presents us with a basic problem. Because rocks are opaque, it is very difficult to see through them and thus it is difficult to know what is the three-dimensional geometry of structures.

This problem can be overcome by using a remote sensing technique known as seismic reflection. This is a geophysical method which is exactly analogous to echo sounding and it is widely used in the petroleum industry. Also several major advances in tectonics have come from recent application of the seismic reflection in academic studies. I'm not going to teach you geophysics, but every modern structural geologist needs to know something about seismic reflection profiling.

Lets examine the simple case of making an echo first to see what the important parameters are.



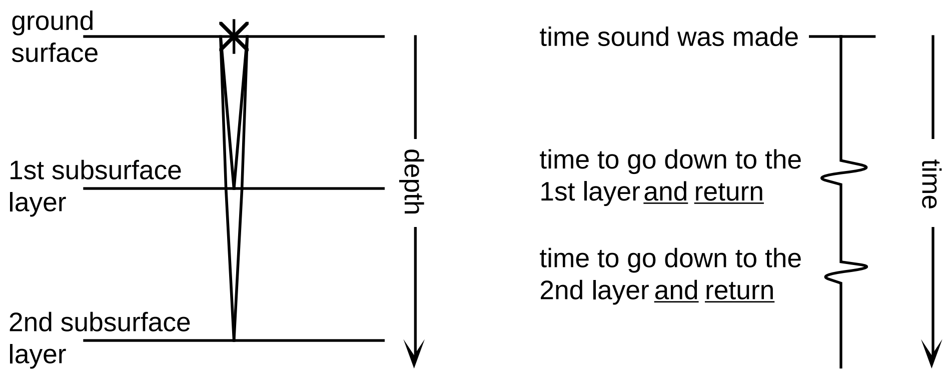
Why do you get a reflection or an echo? You get an echo because the densities and sound velocities of air and rock are very different. If they had the same density and velocity, there would be no echo. More specifically

$$velocity = \mathbf{V} = \sqrt{\frac{E}{\rho}}$$ (E = Young's modulus)

and

$$\text{reflection coefficient} = R = \frac{\text{amplitude of reflected wave}}{\text{amplitude of incident wave}} = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$$

In seismic reflection profiling, what do you actually measure?



The above illustration highlights three important things about seismic reflection profiling:

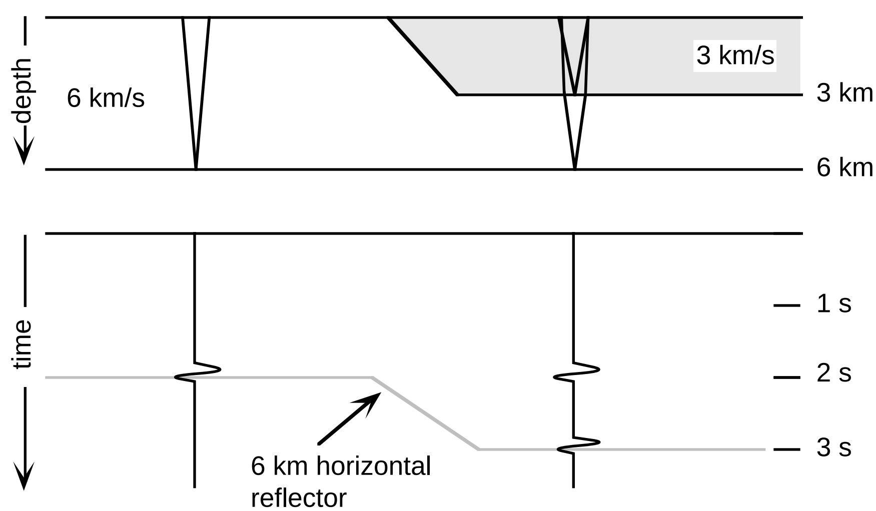
1. Measure time, not depth,

2. The time recorded is round trip or two-way time, and

3. To get the depth, we must know the velocity of the rocks.

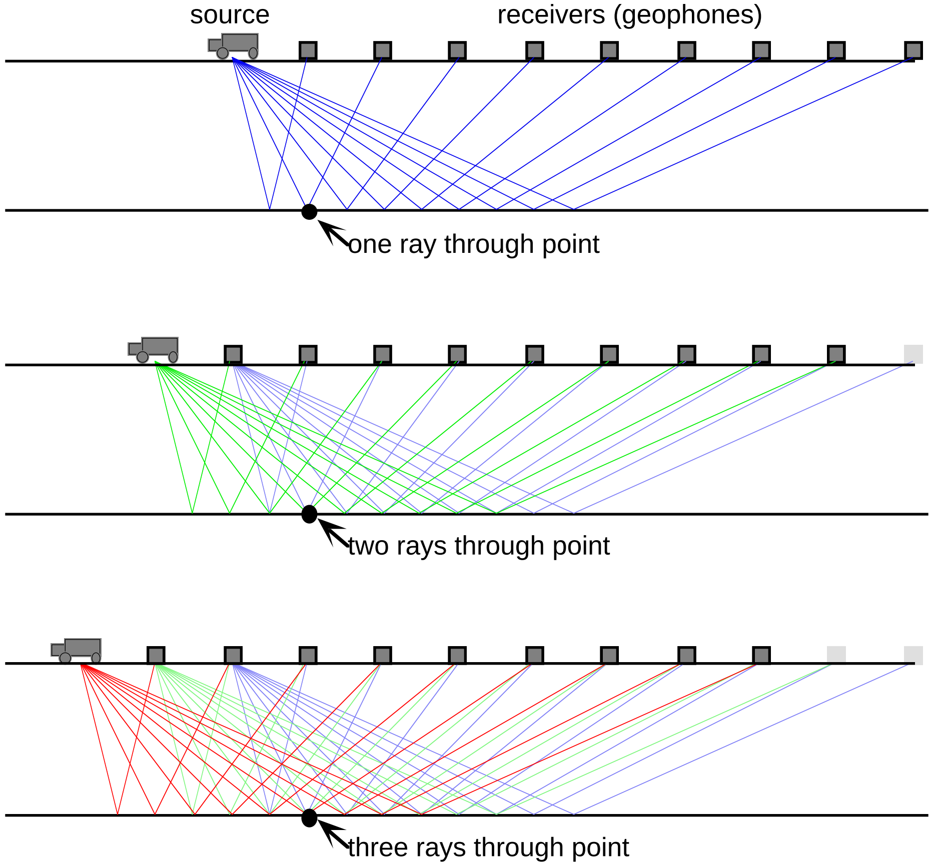
Velocities of rocks in the crust range between about 2.5 km/s and 6.8 km/s. Most sedimentary rocks have velocities of less than 6 km/s. These are velocities of P-waves or compressional waves, not shear waves. Most seismic reflection surveys measure P- not S-waves.

Seismic reflection profiles resemble geologic cross-sections, but they are not. They are distorted because rocks have different velocities. The following diagram illustrates this point.



3.2 Common Depth Point (CDP) Method

In the real earth, the reflectivity at most interfaces is very small, R ≈ 0.01, and the reflected energy is proportional to R^2. Thus, at most interfaces ~99.99% of the energy is transmitted and 0.01% is reflected. This means that your recording system has to be able to detect very faint signals coming back from the subsurface.

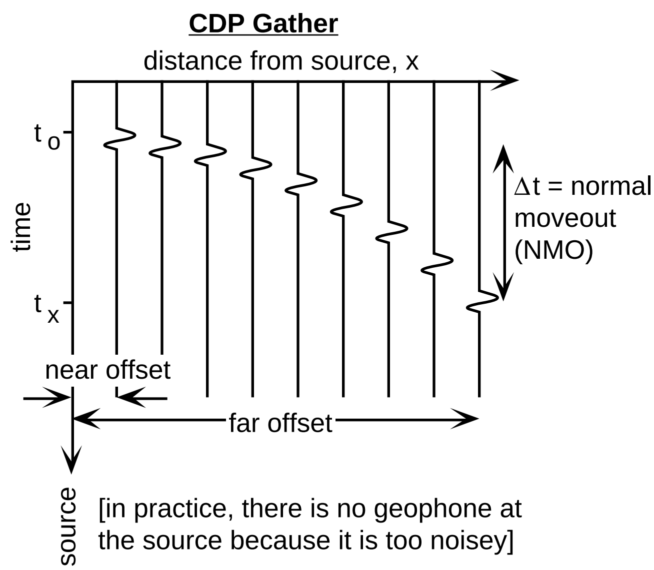


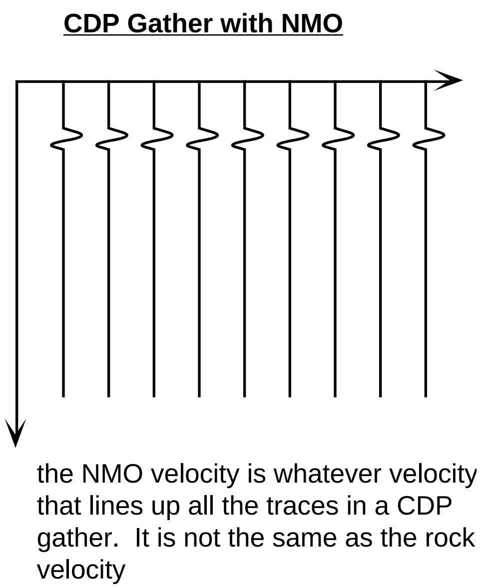
The black dot, and each point on the reflector with a ray going through it, is a common depth point. Notice that there are twice as many CDPs as there are stations on the ground (where the geophones are). That is, there is a CDP directly underneath each station and a CDP half way between each station (hence the name “common midpoint”)

Also, in a complete survey, the number of traces through each midpoint will be equal to one half the total number of active stations at any one time. [This does not include the ends of the lines where there are fewer traces, and it also assumes that the source moves up only one station at a time.] The number of active stations is determined by the number of channels in the recording system. Most modern seismic reflection surveys use at least 96 (and sometimes -- but not often -- as many as 1024 channels), so that the number of traces through any one CDP will be 48.

This number is the data redundancy, of the fold of the data. For example, 24 fold or 2400% means that each depth point was sampled 24 times. Sampling fold in a seismic line is the same thing as the "over-sampling" which you see advertised in compact disk players.

Before the seismic reflection profile can be displayed, there are several intermediate steps. First, all of the traced through the same CDP have to be gathered together. Then you have to determine a set of velocities, known as stacking or NMO velocities, which will correct for the fact that each ray through a CDP has a path of a different length. These velocities should line up all of the individual "blips" corresponding to a single reflector on adjacent traces





The relation between the horizontal offset, x, and the time at which a reflector appears at that offset, tx, is:

$$t_x^2 = t_0^2 + \frac{x^2}{V_{stacking}^2}$$

or

$$\Delta t = t_x - t_0 = \left(t_0^2 + \frac{x^2}{V_{stacking}^2} \right)^{\frac{1}{2}} - t_0$$

<u>If</u> you have a very simple situation in which all of your reflections are flat and there are only

vertical velocity variations (i.e. velocities do not change laterally), then you can calculate the rock interval velocities from the stacking velocities using the Dix equation:

$$V_{i12} = \left(\frac{V_{st2}^2 t_2 - V_{st1}^2 t_1}{t_2 - t_1} \right)^{\frac{1}{2}}$$

where V_{i12} is the interval velocity of the layer between reflections 1 and 2, V_{st1} is the stacking velocity of reflection 1, t_1 is the two way time of reflection 1, etc. The interval velocity is important because, to convert from two-way time to depth, we must know the interval, not the stacking, velocity.

Once the correction for normal moveout is made, we can add all of the traces together, or stack them. This is what produces the familiar seismic reflection profiles.

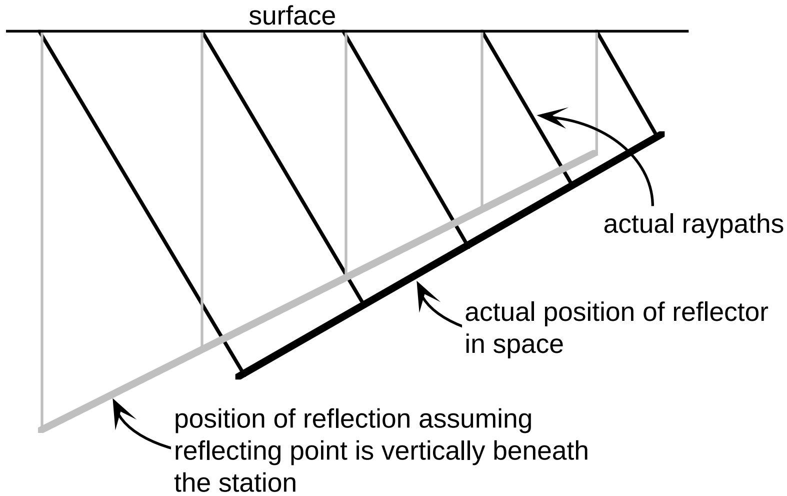
Processing seismic data like this is simple enough, but there are huge amounts of data involved. For example a typical COCORP profile is 20 s long, has a 4 ms digital sampling rate (the time interval between numbers recorded), and is 48 fold. In a hundred station long line, then, we have

$$\frac{(200 \text{ CDPs})(48 \text{ sums})(20 \text{ s})}{\dfrac{0.004 \text{ s}}{\text{data sample}}} = 48 \times 10^6 \text{ data samples}.$$

For this reason, the seismic reflection processing industry is one of the largest users of computers in the world!

3.3 Migration

The effect of this type of processing is to make it look like the source and receiver coincide (e.g. having 48 vertical traces directly beneath the station). Thus, all reflections are plotted as if they were vertically beneath the surface. This assumption is fine for flat layers, but produces an additional distortion for dipping layers, as illustrated below.



Note that the affect of this distortion is that all dipping reflections are displaced down-dip and have a shallower dip than the reflector that produced them. The magnitude of this distortion is a function of the dip of the reflector and the velocity of the rocks.

The process of migration corrects this distortion, but it depends on well-determined velocities and on the assumption that all reflections are in the plane of the section (see “sideswipe”, below). A migrated section can commonly be identified because it has broad “migration smiles” at the bottom and edges. Smiles within the main body of the section probably mean that it has been “over-migrated.”

3.4 Resolution of Seismic Reflection Data

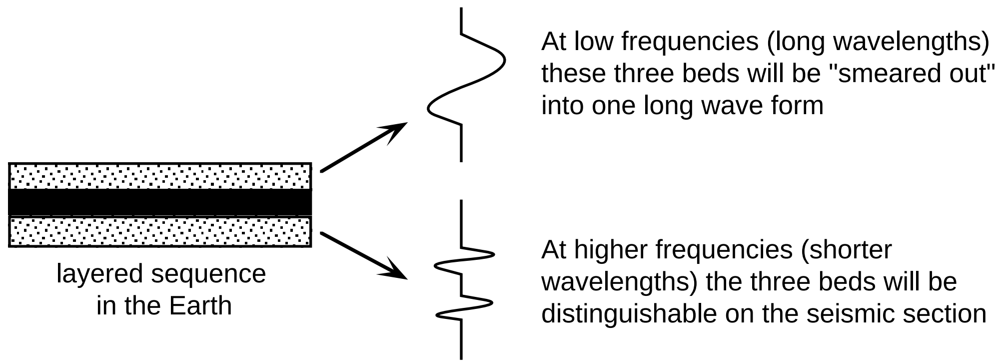
The ability of a seismic reflection survey to resolve features in both horizontal and vertical directions is a function of wavelength:

$$\lambda = \text{velocity} / \text{frequency}.$$

Wavelength increases with depth in the Earth because velocity increases and frequency decreases. Thus, seismic reflection surveys lose resolution with increasing depth in the Earth.

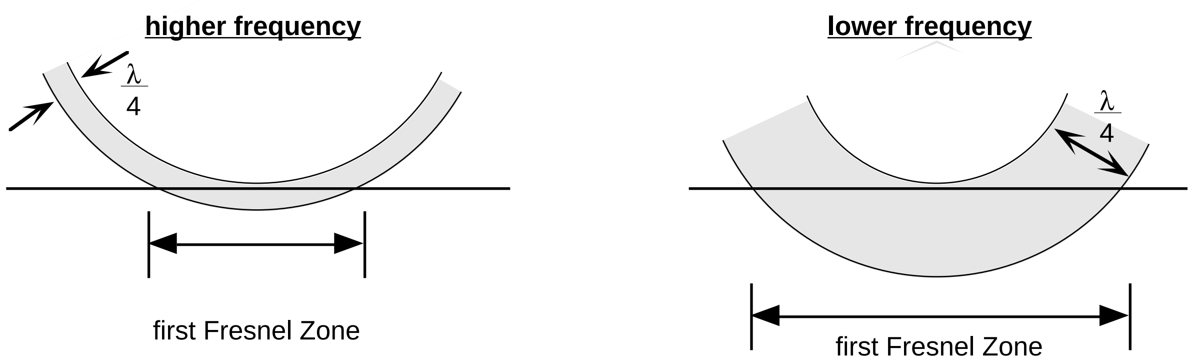
3.4.1 Vertical Resolution

Generally, the smallest (thinnest) resolvable features are 1/4 to 1/8 the dominant wavelength:



3.4.2 Horizontal Resolution

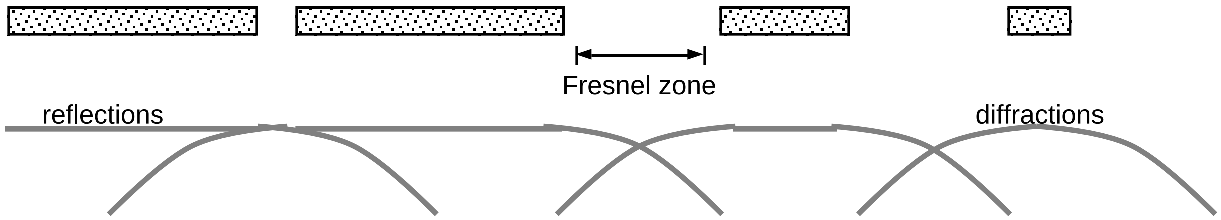
The horizontal resolution of seismic reflection data depends on the Fresnel Zone, a concept which should be familiar to those who have taken optics. The minimum resolvable horizontal dimensions are equal to the first Fresnel zone, which is defined below.



Because frequency decreases with depth in the crust, seismic reflection profiles will have greater horizontal resolution at shallower levels.

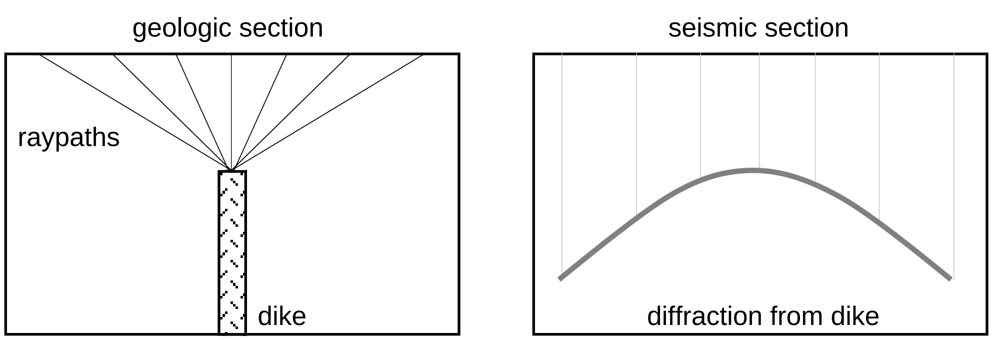
At 1.5 km depth with typical frequencies, the first Fresnel Zone is ~300 m. At 30 km depth, it is about 3 km in width.

Consider a discontinuous sandstone body. The segments which are longer than the first Fresnel Zone will appear as reflections, whereas those which are shorter will act like point sources. Point sources and breaks in the sandstone will generate diffractions, which have hyperbolic curvature:

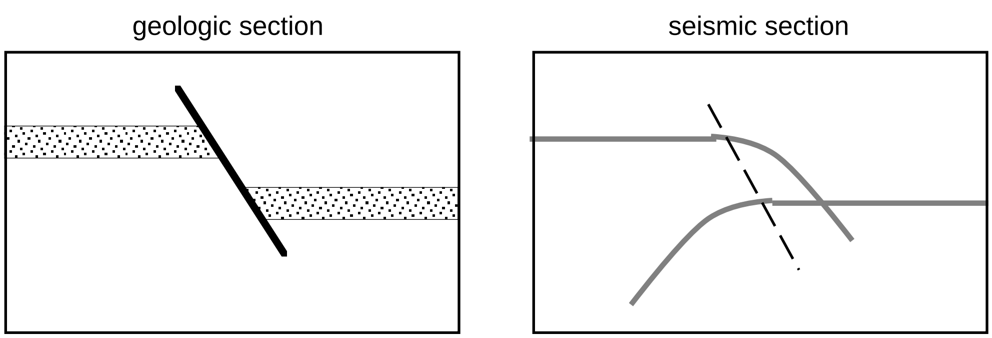


3.5 Diffractions

Diffractions may look superficially like an anticline but they are not. They are extremely useful, *especially because* *seismic reflection techniques are biased toward gently dipping layers and do not image directly steeply dipping or vertical features*. Diffractions help you to identify such features. For example, a vertical dike would not show up directly as a reflection but you could determine its presence by correctly identifying and interpreting the diffractions from it:



High-angle faults are seldom imaged directly on seismic reflection profiles, but they, too, can be located by finding the diffractions from the truncated beds:



The shape and curvature of a diffraction is dependent on the velocity. At faster velocities, diffractions become broader and more open. Thus at great depths in the crust, diffractions may be very hard to distinguish from gently dipping reflections.

3.6 Artifacts

The seismic reflection technique produces a number of artifacts -- misleading features which are easily misinterpreted as real geology -- which can fool a novice interpreted. A few of the more common "pitfalls" are briefly listed below.

3.6.1 Velocity Pullup/pulldown

We have already talked about this artifact when we discussed the distortion due to the fact that seismic profiles are plotted with the vertical dimension in time, not depth. When you have laterally varying velocities, deep horizontal reflectors will be pulled up where they are overlain locally by a high velocity body and will be pushed down by a low velocity body (as in the example on page 2).

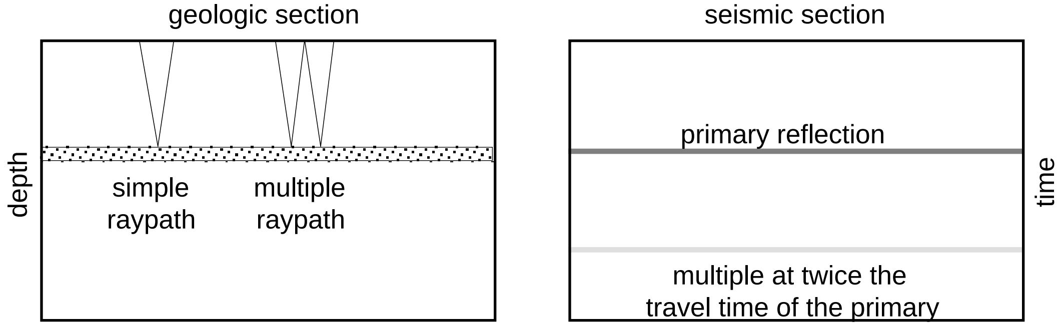
3.6.2 Multiples

Where there are very reflective interfaces, you can get multiple reflections, or multiples, from those interfaces. The effective reflectivity of multiples is the product of the reflectivity of each reflecting interface. For simple multiples (see below) then,

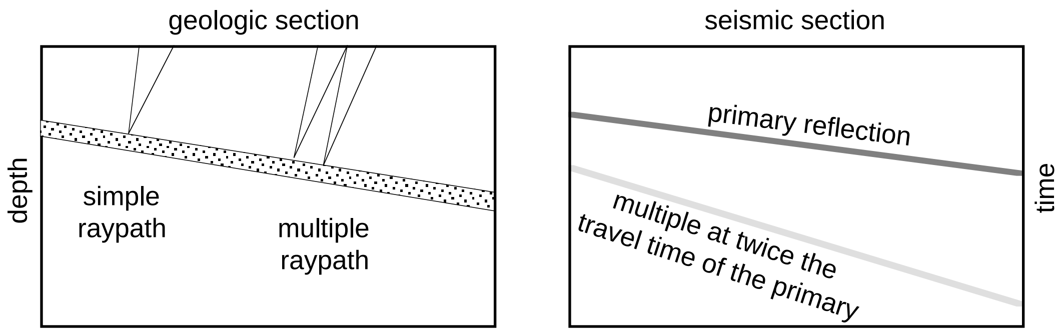
$$R_{multiple} = R^2_{primary}.$$

If the primary reflector has a reflection coefficient of 0.01 then the first multiple will have an effective reflection coefficient of 0.0001. In other words, multiples are generally only a problem for highly reflective interfaces, such as the water bottom in the case of a marine survey or particularly prominent reflectors in sedimentary basins (e.g. the sediment-basement interface).

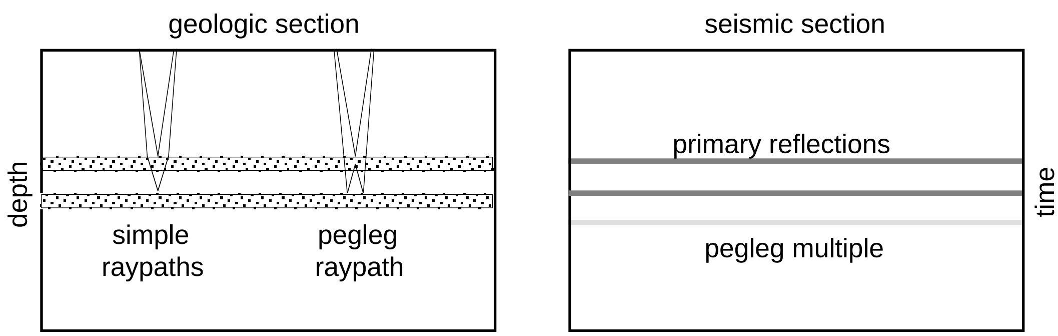
Multiple from a flat layer:



Multiple from a dipping layer (note that the multiple has twice the dip of the primary):

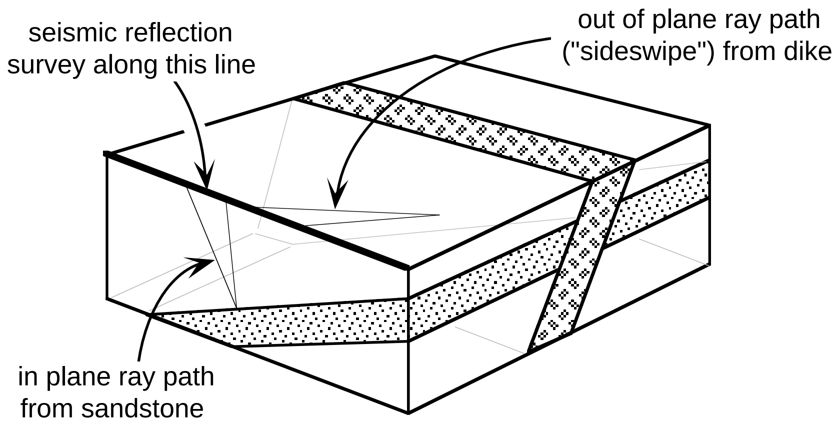


Pegleg multiples:



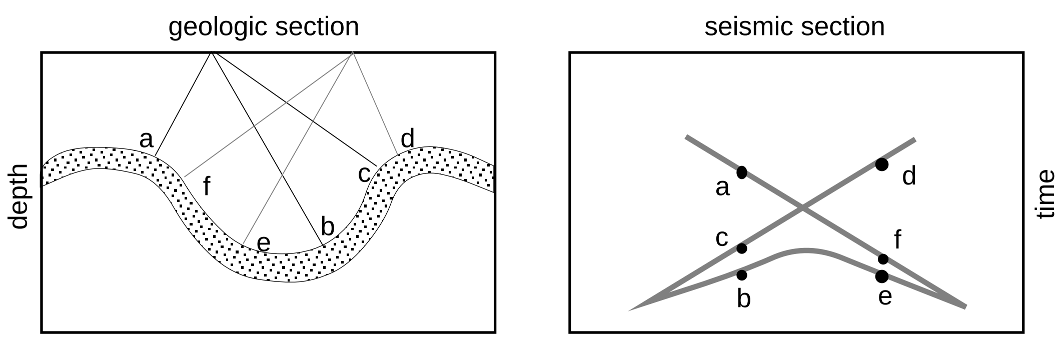
3.6.3 Sideswipe

In seismic reflection profiling, we assume that all the energy that returns to the geophones comes from within the vertical plane directly beneath the line of the profile. Geology is inherently three-dimensional so this need not be true. Even though geophones record only vertical motions, a strong reflecting interface which is out-of-the-plane can produce a reflection on a profile, as in the case illustrated below.



Reflections from out of the plane is called sideswipe. Such reflections will cross other reflections and will not migrate out of the way. (Furthermore they will migrate incorrectly because in migration, we assume that there has been no sideswipe!) The main way of detecting sideswipe is by running a sufficient number of cross-lines and tying reflections from line to line. Sideswipe is particularly severe where seismic lines run *parallel* to the structural grain.

3.6.4 Buried Focus



Tight synclines at depth can act like concave mirrors to produce an inverted image quite unlike the actual structure. Although the geological structure is a syncline, on the seismic profile it looks like an anticline. Many an unhappy petroleum geologist has drilled a buried focus hoping to find an anticlinal trap! The likelihood of observing a buried focus increases with depth because more and more open structures will produce the focus. A good migration will correct for buried focus.

3.6.5 Others

- reflected refractions
- reflected surface waves
- spatial aliasing

LECTURE 4 — INTRODUCTION TO DEFORMATION

4.1 Introduction

In this part of the course, we will first lay out the mechanical background of structural geology before going on to explain the structures, themselves. As stated in the first lecture, what we, as geologists, see in the field are deformed rocks. We do not see the forces acting on the rocks today, and we certainly do not see the forces which produced the deformation in which we are interested. Thus, deformation would seem to be an obvious starting point in our exploration of structural geology.

There is a natural hierarchy to understanding how the Earth works from a structural view point:

- geometry
- kinematics
- mechanics ("dynamics")

We have briefly addressed some topics related to geometry and how we describe it; the lab part of this course deals almost exclusively with geometric methods.

4.2 Kinematics

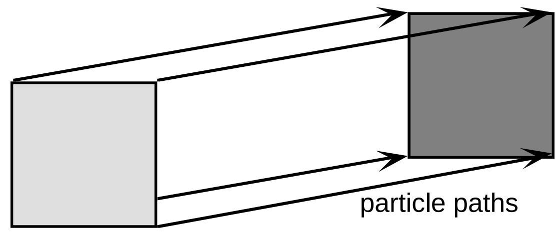
"Kinematic analysis" means reconstructing the movements and distortions that occur during rock deformation. Deformation is the process by which the particles in the rock rearrange themselves from some initial position to the final position that we see today. The components of deformation are:

Rigid body deformation
Translation
Rotation

Non-rigid Body deformation (STRAIN)
Distortion
Dilation

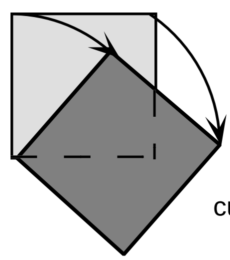
4.2.1 Rigid Body Deformations

Translation = movement of a body without rotation or distortion:



in translation, all of the particle paths are straight, constant length, and parallel to each other.

Rotation = rotation of the body about a common axis. In rotation, the particle paths are curved and concentric.



The sense of rotation depends on the position of the viewer. The rotation axis is defined as a vector pointing in the direction that the viewer is looking:

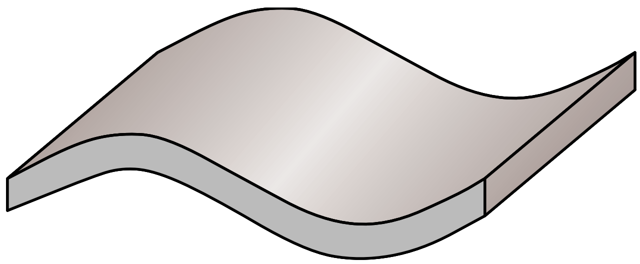
Translation and rotation commonly occur at the same time, but mathematically we can treat them completely separately

4.2.2 Strain (Non-rigid Body Deformation)

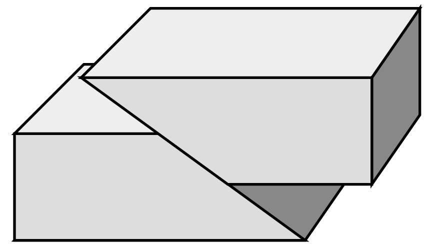
Four very important terms:

Continuous -- strain properties vary smoothly throughout the body with no abrupt changes.

Discontinuous -- abrupt changes at surfaces, or breaks in the rock



fold is <u>continuous</u>

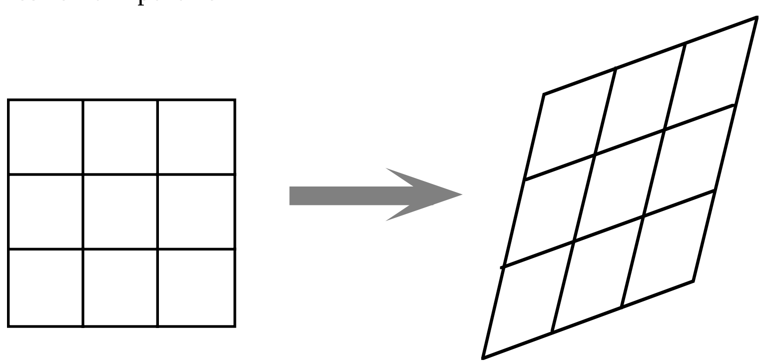


fault is <u>discontinuous</u>

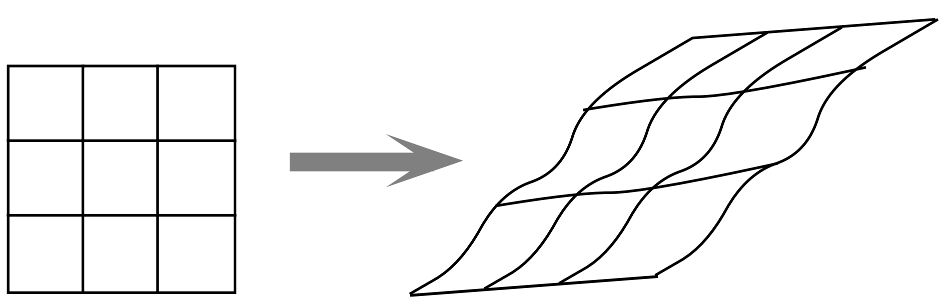
Homogeneous -- the properties of strain are identical throughout the material. Each particle of material is distorted in the same way. There is a simple test if the deformation is homogeneous:

1. Straight lines remain straight

2. Parallel lines remain parallel



<u>Heterogeneous</u> --the type and amount of strain vary throughout the material, so that one part is more deformed than another part.



This diagram does not fit the above test so it is heterogeneous. You can see that a fold would be a heterogeneous deformation.

4.2.3 Continuum Mechanics

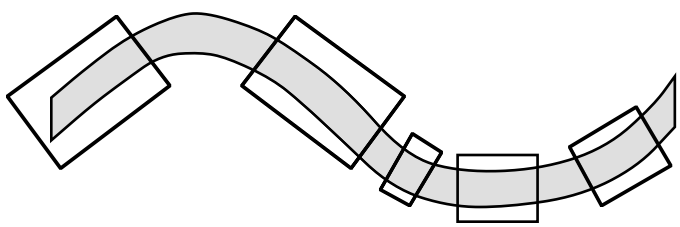
Mathematically, we really only have the tools to deal with continuous deformation. Thus, the study of strain is a branch of continuum mechanics. This fancy term just means “the mechanics of materials with smoothly varying properties.” Such materials are called “continua.”

Right away, you can see a paradox: Geological materials are full of discontinuous features: faults, cracks, bedding surfaces, etc. So, why use continuum mechanics?

1. The mathematics of discontinuous deformation is far more difficult.

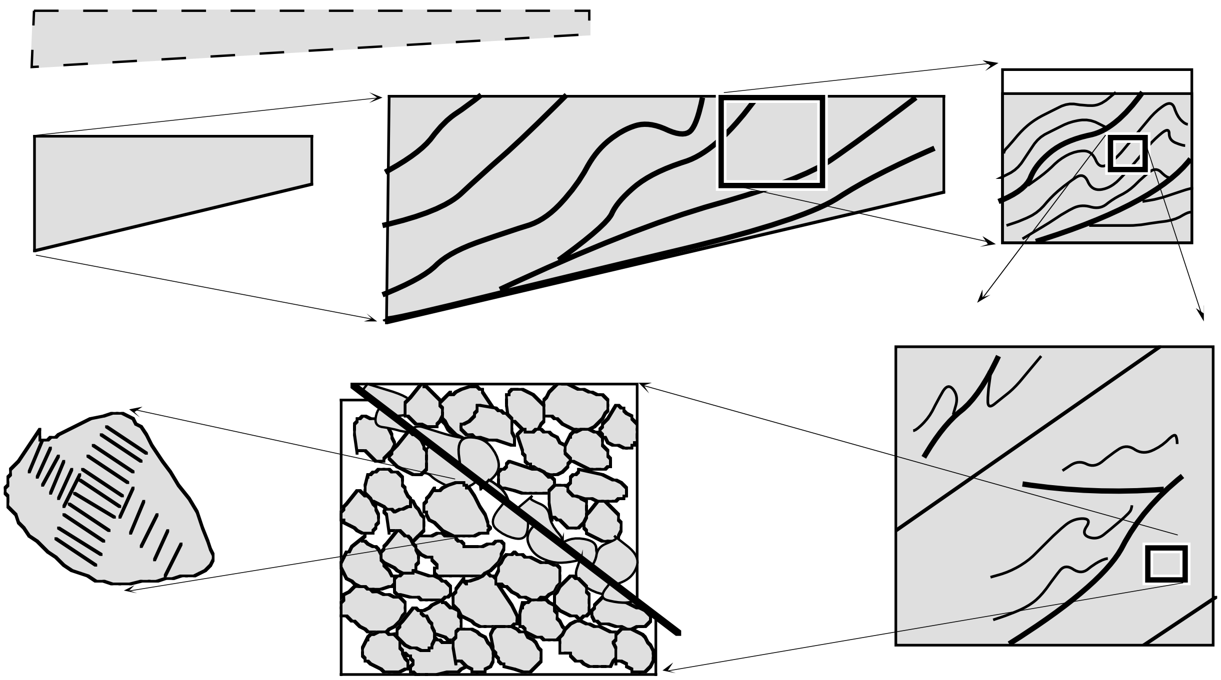
2. At the appropriate **scale** of observation, continuum mechanics is an adequate approximation.

We also analyze homogeneous strain because it is easier to deal with. To get around the problem of heterogeneous deformation, we apply the concept of structural domains. These are regions of more-or-less homogeneous deformation within rocks which, at a broader scale, are heterogeneous. Take the example of a fold:

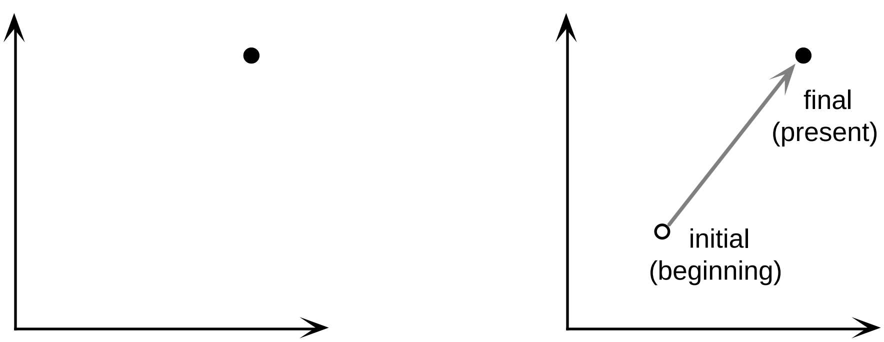


The approximations that we make in order to analyze rocks as homogeneous and continuous again depend on the scale of observation and the vantage point of the viewer.

Let’s take a more complex, but common example of a thrust-and-fold belt:



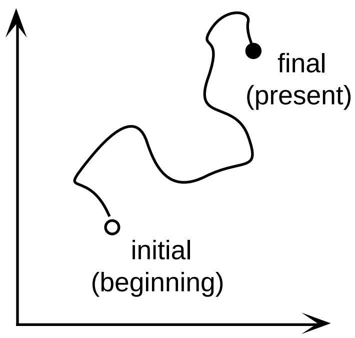
4.2.4 Four Aspects of a Deforming Rock System:



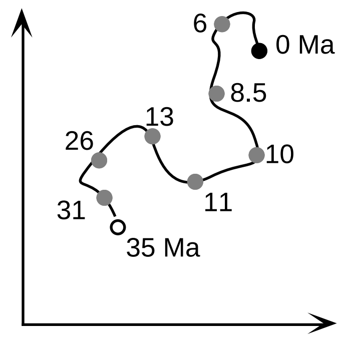
1. Position today **2. Displacement**

Position today is easy to get. It's just the latitude and longitude, or whatever convenient measure you want to use (e.g. "25 km SW of Mt. Marcy" etc.).

The displacement is harder to get because we need to know both the initial and the final positions of the particle. The line which connects the initial and final positions is the displacement vector, or what we called earlier, the particle path.

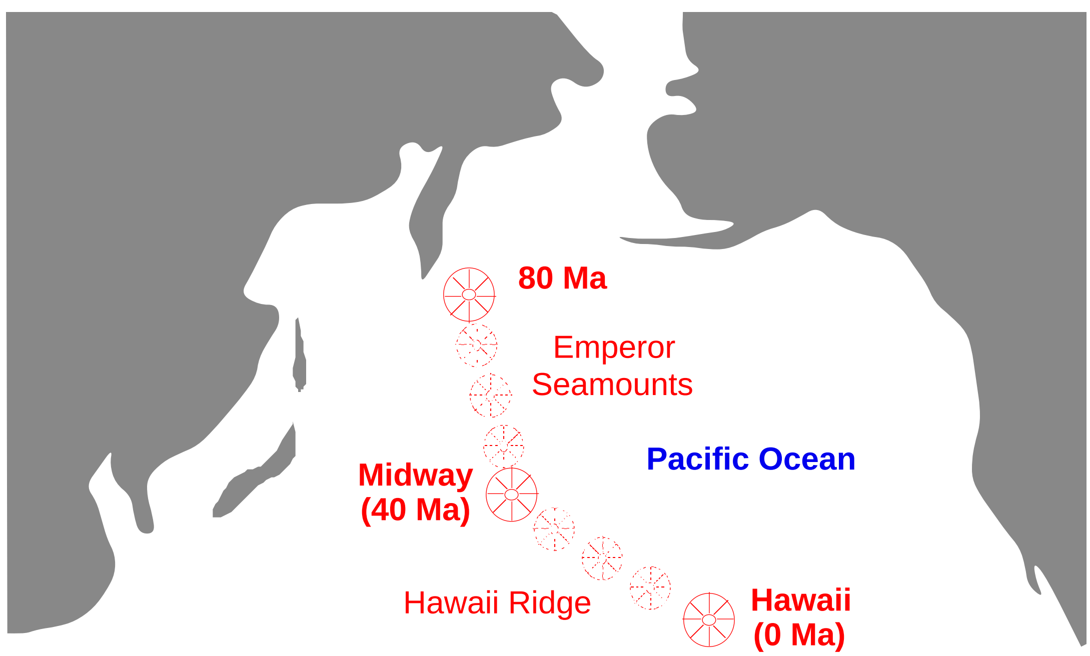


3. Path



3. Dated Path

Ideally, of course, we would like to be able to determine the dated path in all cases, but this is usually just not possible because we can't often get that kind of information out of the earth. There are some cases, though:



4.3 Measurement of Strain

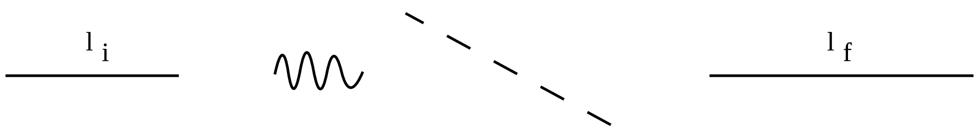
There are three types of things we can measure:

1. Changes in the lengths of <u>lines</u>,

2. Changes in <u>angles</u>

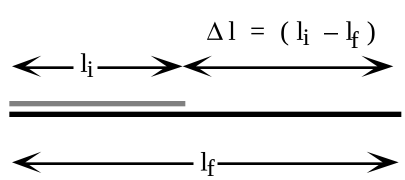
3. Changes in <u>volume</u>

In all cases, we are comparing a final state with an initial state. What happens between those two states is not accounted for (i.e. the displacement path, #3 above, is not accounted for).



4.3.1 Change in Line Length:

<u>Extension</u>:



we define extension (elongation) $$e \equiv \frac{\Delta l}{l_i} = \frac{\left(l_f - l_i\right)}{l_i} = \frac{l_f}{l_i} - 1 \tag{4.1}$$

shortening is negative

<u>Stretch</u>: $$S \equiv \frac{l_f}{l_i} = 1 + e \tag{4.2}$$

<u>Quadratic elongation</u>: $$\lambda = S^2 = (1 + e)^2 \tag{4.3}$$

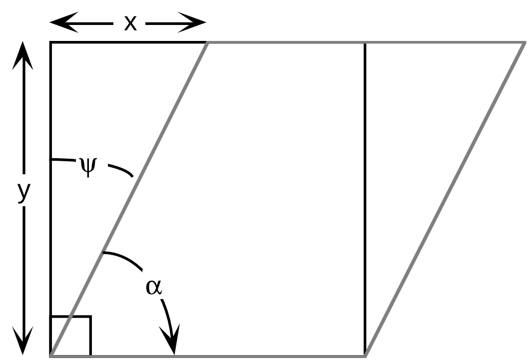
if $\lambda = 1$ then no change

if $\lambda < 1$ then shortening

if $\lambda > 1$ then extension

$\lambda \geq 0$ because it is a function of S^2. It will only be 0 if volume change reduces l_f to zero.

4.3.2 Changes in Angles:



There are two ways to look at this deformation:

1. Measure the change in angle between two originally perpendicular lines:

   change in angle = 90 - α = ψ ≡ <u>angular shear</u>

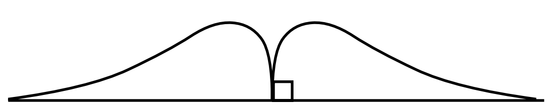
2. Look at the displacement, x, of a particle at any distance, y, from the origin (a particle which does not move):

$$\frac{x}{y} = \gamma \quad \equiv \textbf{\underline{shear strain}} \tag{4.4}$$

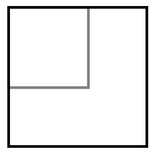
The relationship between these two measures is a simple trig function:

$$\gamma = \tan \psi \tag{4.5}$$

γ and ψ are very useful geologically because there are numerous features which we know were originally perpendicular (e.g. worm tubes, bilaterally symmetric fossils, etc.):



4.3.3 Changes in Volume (Dilation):

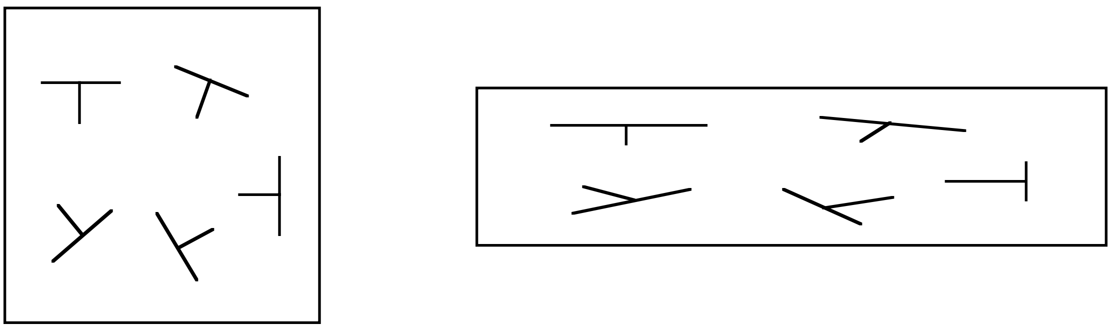


$$\text{Dilation} = \Delta \equiv \frac{\left(V_f - V_i\right)}{V_i} \tag{4.6}$$

LECTURE 5 — STRAIN II: THE STRAIN ELLIPSOID

5.1 Motivation for General 3-D Strain Relations

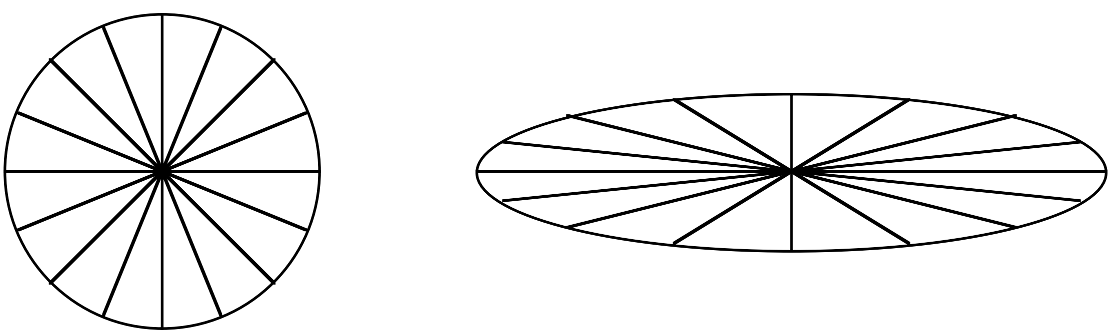
Last class, we considered how to measure the strain of individual lines and angles that had been deformed. Consider a block with a bunch of randomly oriented lines:



Point out how each line and angle change and why.

Well, we now have equations to describe what happens to each individual line and angle, but how do we describe how the body as a whole changes?

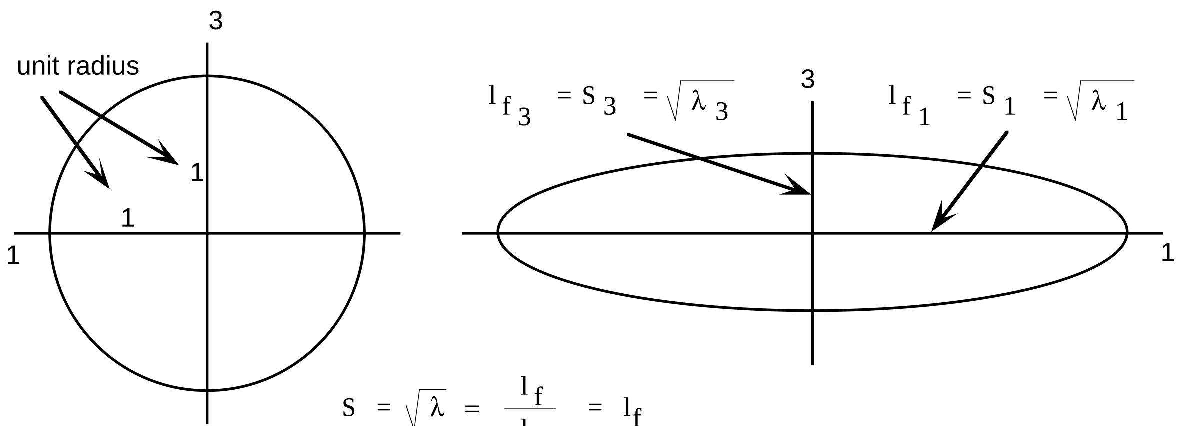
We could mark the body with lines of all different orientations and measure each one -- not very practical in geology. There is, however, a simple geometric object which describes lines of all different orientations but with equal length, a circle:



Any circle that is subjected to homogeneous strain turns into an ellipse. In three dimensions, a sphere turns into an ellipsoid. You'll have to take this on faith right now but we'll show it to be true later on.

5.2 Equations for Finite Strain

Coming back to our circle and family of lines concept, let's derive some equations that describe how any line in the body changes length and orientation.



$$S = \sqrt{\lambda} = \frac{l_f}{l_i} = l_f$$

[sometimes you'll see the 1 and 3 axes referred to as the "X" and "Z" axes, respectively]

The general equation for a circle is: $x^2 + z^2 = 1$,

and for an ellipse:

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \tag{5.1}$$

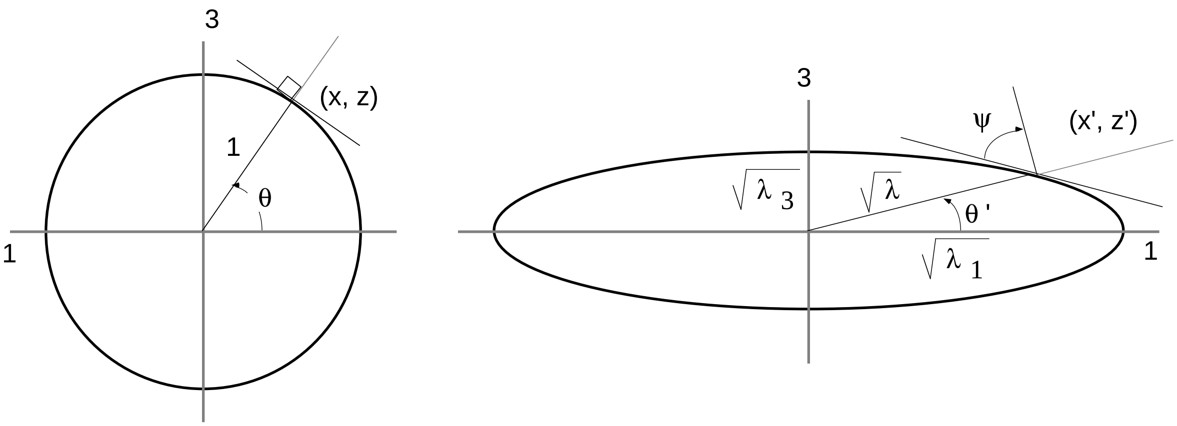
where a & b are the major and minor axes.

So, the equation of the strain ellipse is:

$$\frac{x^2}{\lambda_1} + \frac{z^2}{\lambda_3} = 1 \tag{5.2}$$

5.3 Extension of a Line

Now, let's determine the strain of any line in the deformed state:



From the above, you can see that:

$$z' = \sqrt{\lambda}\sin\theta' \qquad \text{and} \qquad x' = \sqrt{\lambda}\cos\theta' \tag{5.3}$$

Substituting into the strain ellipse equation (5.2), we get

$$\frac{\lambda\sin^2\theta'}{\lambda_3} + \frac{\lambda\cos^2\theta'}{\lambda_1} = 1. \tag{5.4}$$

Dividing both sides by λ, yields:

$$\frac{\sin^2\theta'}{\lambda_3} + \frac{\cos^2\theta'}{\lambda_1} = \frac{1}{\lambda}\ . \tag{5.5}$$

We can manipulate this equation to get a more usable form by using some standard trigonometric double angle formulas:

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha\ . \tag{5.6}$$

Cranking through the substitutions, and rearranging:

$$\frac{\lambda_3 + \lambda_1 + (\lambda_3 - \lambda_1)\cos 2\theta'}{2\lambda_1\lambda_3} = \frac{1}{\lambda}\ . \tag{5.7}$$

If we let

$$\lambda' = \frac{1}{\lambda}\ ,\ \ \lambda_1' = \frac{1}{\lambda_1}\ ,\ \text{ and }\ \lambda_3' = \frac{1}{\lambda_3}\ ,$$

then

$$\frac{(\lambda_3' + \lambda_1')}{2} - \frac{(\lambda_3' - \lambda_1')}{2}\cos 2\theta' = \lambda'\ . \tag{5.8}$$

5.4 Shear Strain

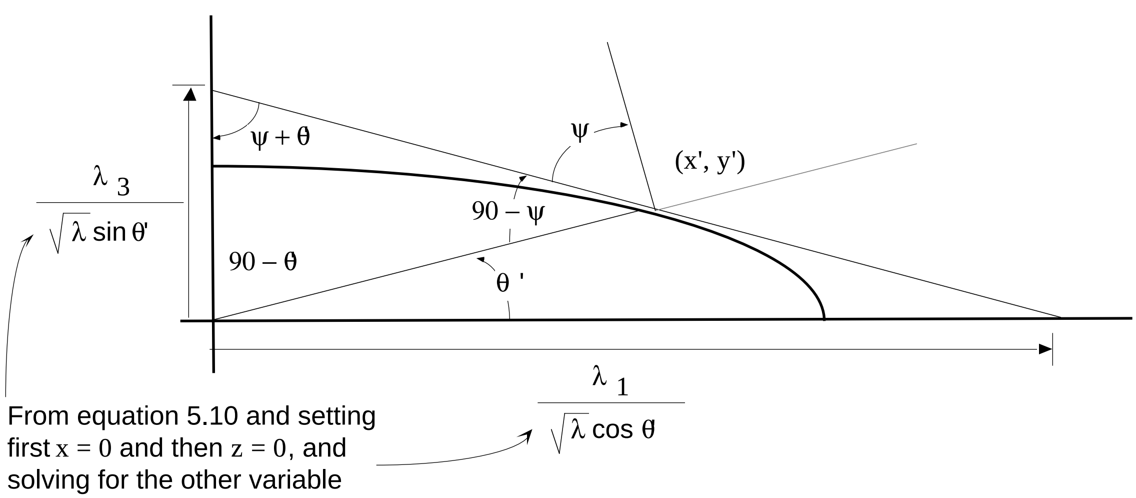
To get the shear strain, you need to know the equation for the tangent to an ellipse:

$$\frac{x\mathrm{x}'}{\lambda_1} + \frac{z\mathrm{z}'}{\lambda_3} = 1\ . \tag{5.9}$$

Substituting equations 5.3 (page 44) into 5.9:

$$\frac{x\sqrt{\lambda}\cos\theta'}{\lambda_1} + \frac{z\sqrt{\lambda}\sin\theta'}{\lambda_3} = 1\ , \tag{5.10}$$

we can solve for the intercepts of the tangent:



From the trigonometry of the above triangle (from here, it can be solved in a lot of different ways):

$$\tan(\psi + \theta') = \frac{\tan\psi + \tan\theta'}{1 - \tan\psi\tan\theta'} = \frac{\lambda_1}{\lambda_3}\tan\theta'\ \ .$$

Recall that: $\tan\psi = \gamma$.

Lots of substitutions later:

$$\gamma = \frac{(\lambda_1 - \lambda_3)\sin\theta'\cos\theta'}{\lambda_3\cos^2\theta' + \lambda_1\sin^2\theta'}.$$

The denominator is just $\frac{\lambda_1\lambda_3}{\lambda}$, which you get by multiplying eqn. 5.4 by $\lambda_1\lambda_3$ and dividing by λ. Eventually, you get

$$\frac{\gamma}{\lambda} = \frac{1}{2}\left(\frac{1}{\lambda_3} - \frac{1}{\lambda_1}\right)\sin 2\theta'.$$

and with the same reciprocals as we used before (top of page 45):

$$\gamma' = \frac{\gamma}{\lambda} = \frac{(\lambda_3' - \lambda_1')}{2}\sin 2\theta' \qquad (5.11)$$

Next time, we'll see what all this effort is useful for…

LECTURE 6 — STRAIN III: MOHR ON THE STRAIN ELLIPSOID

6.1 Introduction

Last time, we derived the fundamental equations for the strain ellipse:

$$\lambda' = \frac{(\lambda_3' + \lambda_1')}{2} - \frac{(\lambda_3' - \lambda_1')}{2}\cos 2\theta' \tag{6.1}$$

and

$$\gamma' = \frac{\gamma}{\lambda} = \frac{(\lambda_3' - \lambda_1')}{2}\sin 2\theta' \tag{6.2}$$

These equations are of the same form as the parametric equations for a circle:

$$x = c - r\cos\alpha$$

$$y = r\sin a ,$$

where the center of the circle is located at (c, 0) on the X-axis and the circle has a radius of "r". Thus, the above equations define a circle with a center at

$$(c,\ 0) = \left(\frac{\lambda_3' + \lambda_1'}{2},\ 0\right)$$

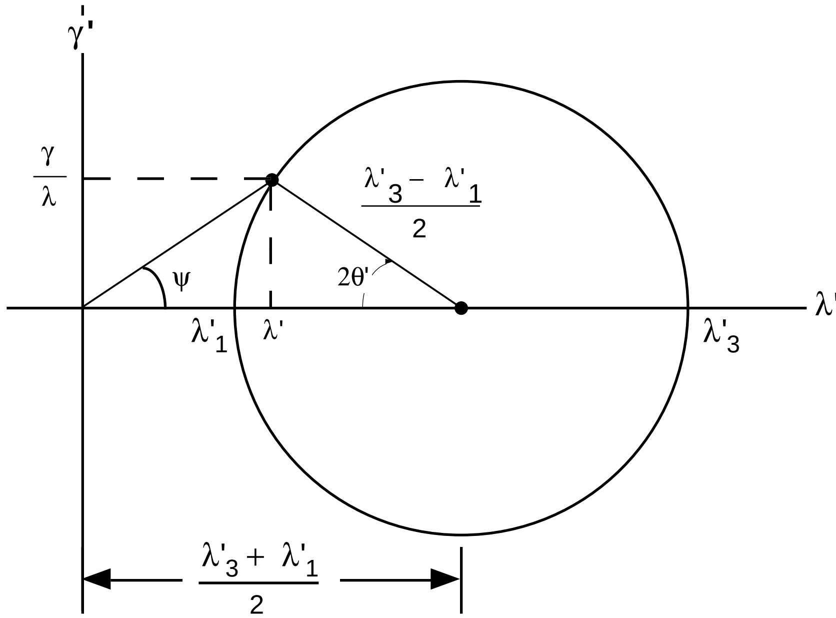
and radius

$$r = \left(\frac{\lambda_3' - \lambda_1'}{2}\right).$$

These equations define the <u>Mohr's Circle for finite strain</u>.

6.2 Mohr's Circle For Finite Strain

The Mohr's Circle is a graphical construction devised by a German engineer, Otto Mohr, around the turn of the century. It actually is a graphical solution to a two dimensional tensor transformation, which we mentioned last time, and can be applied to any symmetric tensor. We will see the construction again when we talk about stress. But, for finite strain, it looks like:



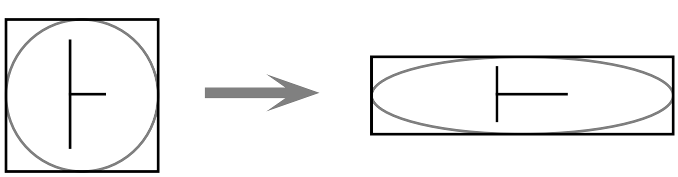
You can prove to yourself with some simple trigonometry that the angle between the λ'-axis and a line from the origin to the point on the circle that represents the strain of the line really is ψ:

$$\tan\psi = \frac{\gamma'}{\lambda'} = \frac{\gamma/\lambda}{1/\lambda} = \gamma$$

6.3 Principal Axes of Strain

λ_1 and λ_3, the long and short axes of the finite strain ellipse, are known as the **principal axes of strain** because they are the lines which undergo the maximum and minimum amounts of extension. From the Mohr's Circle, we can see a very important property of the principal axes. They are the only two points on the circle that intersect the horizontal axis.

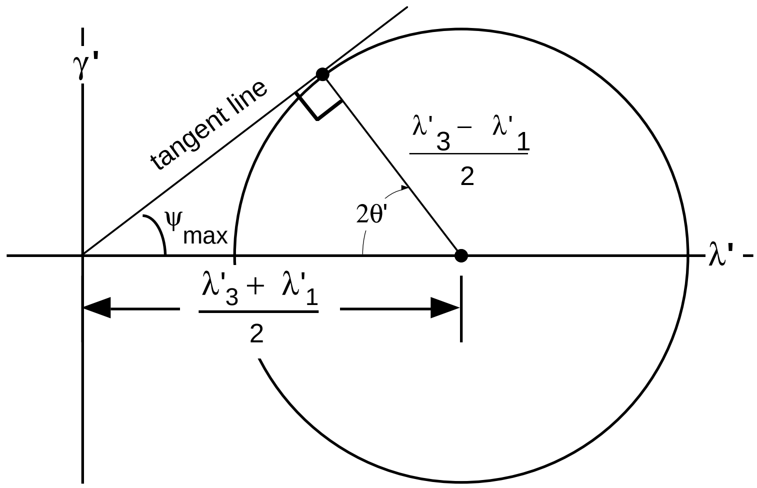
Thus, lines parallel to the principal axes suffer no shear strain or angular shear. All other lines in the body do undergo angular shear.



Lines are perpendicular before and after the deformation because they are parallel to the principal axes

6.4 Maximum Angular Shear

You can also use the Mohr's Circle to calculate the orientation and extension of the line which undergoes the maximum angular shear, ψ_{max}, and shear strain, γ_{max}:



From the geometry above,

$$\sin\psi_{max} = \frac{\lambda'_3 - \lambda'_1}{2} \Big/ \frac{\lambda'_3 + \lambda'_1}{2} = \frac{\lambda'_3 - \lambda'_1}{\lambda'_3 + \lambda'_1},$$

or

$$\psi_{max} = \sin^{-1}\left(\frac{\lambda'_3 - \lambda'_1}{\lambda'_3 + \lambda'_1}\right). \tag{6.3}$$

To get the orientation of the line with maximum angular shear, $\theta'_{\psi max}$:

$$\cos 2\theta'_{\psi_{max}} = \frac{\lambda'_3 - \lambda'_1}{2} \Big/ \frac{\lambda'_3 + \lambda'_1}{2} = \frac{\lambda'_3 - \lambda'_1}{\lambda'_3 + \lambda'_1},$$

or

$$\theta'_{\psi_{max}} = \frac{1}{2}\cos^{-1}\left(\frac{\lambda'_3 - \lambda'_1}{\lambda'_3 + \lambda'_1}\right). \tag{6.4}$$

You could also easily solve this problem by differentiating with respect to θ, and setting it equal to zero:

$$\frac{d\gamma}{d\theta'} = 0 \ .$$

6.5 Ellipticity

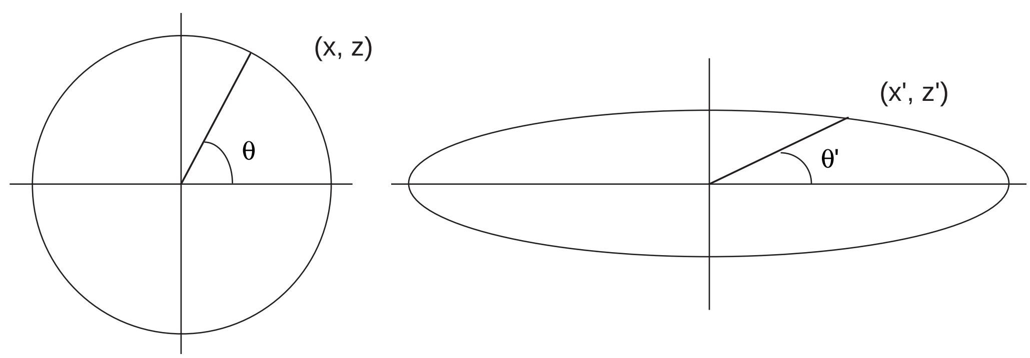
This is a commonly used parameter which describes the aspect ratio (i.e. the ratio of the large and small axes) of the strain ellipse. Basically, it tells you something about the two-dimensional shape of the strain ellipse.

$$R = \frac{(1+e_1)}{(1+e_3)} = \frac{S_1}{S_3} \ . \tag{6.5}$$

Note that, because S_1 is always greater than S_3 (by definition), R is always greater than 1. A circle has an R of 1.

6.6 Rotation of Any Line During Deformation

It is a simple, yet important, calculation to determine the amount that any line has rotated during the deformation:



$$\tan\theta = \frac{z}{x} \qquad \tan\theta' = \frac{z'}{x'}$$

The stretches along the principal axes, 1 and 3, are:

$$S_1 = \sqrt{\lambda_1} = \frac{x'}{x} \quad \Rightarrow \quad x' = x\sqrt{\lambda_1}$$

and

$$S_3 = \sqrt{\lambda_3} = \frac{z'}{z} \quad \Rightarrow \quad z' = z\sqrt{\lambda_3} \ .$$

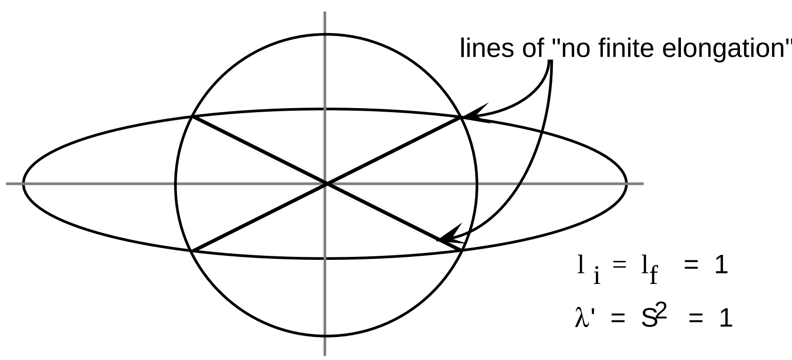
Substituting into the above equations, we get a relation between θ and θ':

$$\tan\theta' = \frac{z\sqrt{\lambda_3}}{x\sqrt{\lambda_1}} = \tan\theta\frac{\sqrt{\lambda_3}}{\sqrt{\lambda_1}} = \tan\theta\frac{S_3}{S_1} = \frac{\tan\theta}{R} \quad . \tag{6.6}$$

The amount of rotation that any line undergoes then is just (θ - θ').

6.7 Lines of No Finite Elongation

In any homogeneous deformation without a volume change, there are two lines which have the same length both before and after the deformation. These are called “**lines of no finite elongation**” (LNFE):



We can solve for the orientations of these two lines by setting the Mohr Circle equation for elongation to 1,

$$\lambda' = \frac{(\lambda_3' + \lambda_1')}{2} - \frac{(\lambda_3' - \lambda_1')}{2}\cos 2\theta' = 1,$$

and solving for θ':

$$\cos 2\theta' = \frac{(\lambda_3' + \lambda_1' - 2)}{(\lambda_3' - \lambda_1')} = 2\cos^2\theta' - 1 \,,$$

and

$$\cos^2\theta' = \frac{(\lambda_3' - 1)}{(\lambda_3' - \lambda_1')} \quad . \tag{6.7}$$

There are alternative forms which use θ instead of θ' and λ instead of λ':

$$\tan^2\theta = \frac{(\lambda_1 - 1)}{(1 - \lambda_3)}$$

and

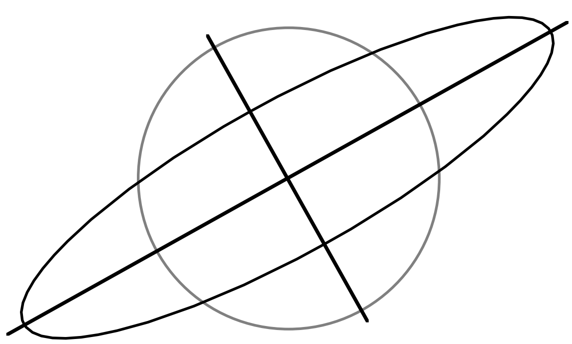
$$\tan^2\theta' = \frac{\lambda_3}{\lambda_1}\frac{(\lambda_1 - 1)}{(1 - \lambda_3)} \quad .$$

LECTURE 7 — STRAIN IV: FINITE VS. INFINITESIMAL STRAIN

Up until now, we've mostly been concerned with describing just the initial and final states of deformed objects. We've only barely mentioned the progression of steps by which things got to their present condition. What we've been studying is **finite strain** -- the total difference between initial and final states. Finite strain can be thought of as the sum of a great number of very small strains. Each small increment of strain is known as Infinitesimal Strain. A convenient number to remember is that an infinitesimal strain is any strain up to about 2%; that is:

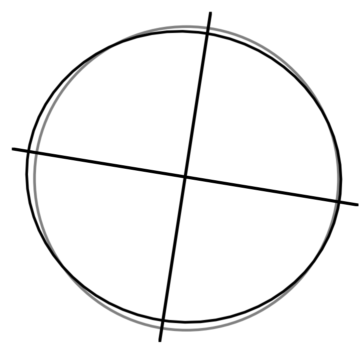
$$e = \frac{l_f - l_i}{l_i} \leq 0.02$$

With this concept of strain, at any stage of the deformation, there are two strain ellipsoids that represent the strain of the rock:



Finite Strain Ellipse

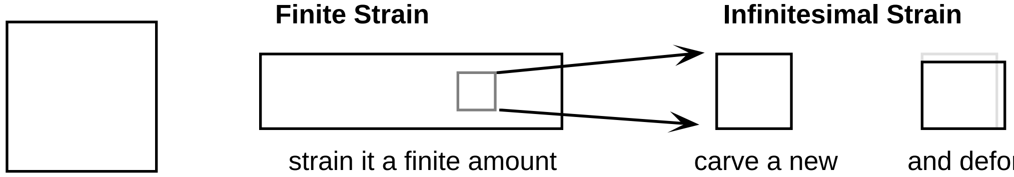
This represents the total deformation from the beginning up until the present.



Infinitesimal Strain Ellipse

This is the strain that the particles will feel in the next instant of deformation

You can look at it this way:



Key aspect of infinitesimal strain:

- The maximum angular shear is always at 45° to the principal axes

7.1 Coaxial and Non-coaxial Deformation

Notice that, in the above drawing, I purposely made the axes of the infinitesimal strain ellipse have a different orientation than those of the finite strain ellipse. Obviously, this is one of two cases -- in the other, the axes would be parallel. This is a very important distinction for understanding deformation:

- Coaxial -- if the axes of the finite and infinitesimal strain ellipses are parallel
- Non-coaxial -- when the axes of finite and infinitesimal are not parallel

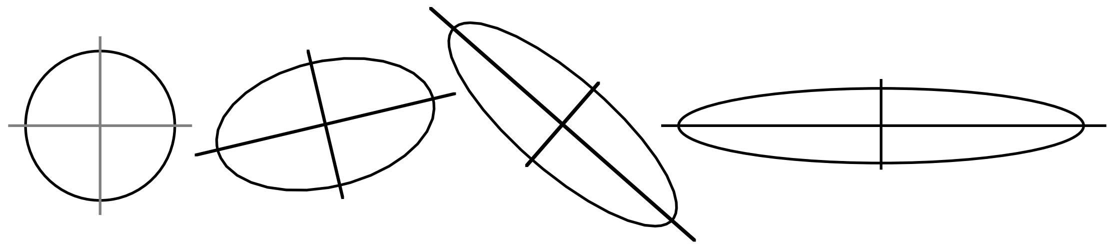
These two terms should not be confused (as they, unfortunately, usually are in geology) with the following two terms, which refer just to finite strain.

- Rotational -- when the axes of the finite strain ellipse are not parallel to their restored configuration in the undeformed, initial state
- Non-rotational -- the axes in restored and final states are parallel

In general in the geological literature, rotational / non-coaxial deformation is referred to as **simple shear** and non-rotational / coaxial deformation is referred to as **pure shear**. The following table may help organize, if not clarify, this concept:

Finite Strain	Infinitesimal Strain
Non-rotational ⇒ pure shear	Coaxial ⇒ progressive pure shear
Rotational ⇒ Simple shear	Non-coaxial ⇒ progressive simple shear

In practice, it is difficult to apply these distinctions, which is why most geologists just loosely refer to pure shear and simple shear. Even so, it is important to understand the distinction, as the following diagram illustrates:



A non-coaxial, non-rotational deformation

7.2 Two Types of Rotation

Be very careful to remember that there are two different types of rotations that we can talk about in deformation:

1. The rotation of the principal axes during the deformation. This occurs only in non-coaxial deformation.

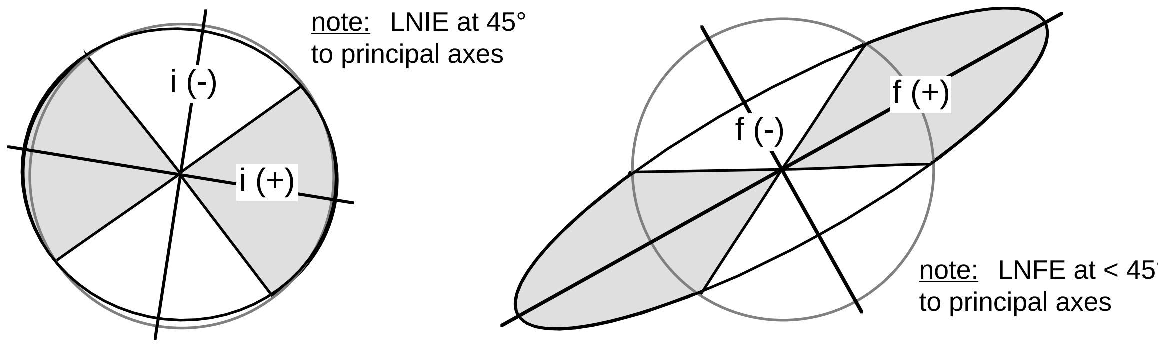
2. The rotation of all other lines in the body besides the principal axes. You can easily calculate this from the equations that we derived in the last two classes (e.g., eqn. 6.6, p. 51). This rotation affects all lines in the body except the principal axes. This rotation has nothing to do with whether or not the deformation is by pure or simple shear.

If we know the magnitudes of the principal axes and the initial or final position of the line, it is always possible to calculate the second type of rotation. Without some external frame of reference, it is impossible to calculate the first type of rotation. In other words, if I have a deformed fossil and can calculate the strain, I still do not know if it got to it's present condition via a coaxial or non-coaxial strain path.

Many a geologist has confused these two types of rotation!!

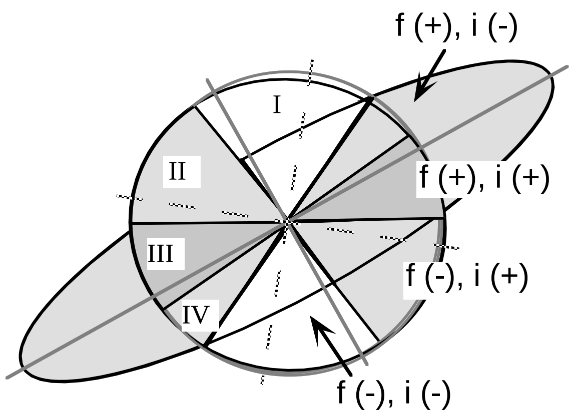
7.3 Deformation Paths

Most geologic deformations involve a non-coaxial strain path. Thus, in general, the axes of the infinitesimal and finite strain ellipsoids will not coincide. In the diagram below, all the lines which are within the shaded area of the infinitesimal strain ellipse ["i(+)"] will become infinitesimally longer in the next tiny increment of deformation; they may still be shorter than they were originally. In the shaded area of the finite strain ellipse ["f(+)"], all of the lines are longer than they started out.



Infinitesimal strain **Finite Strain**

Thus, the history of deformation that any line undergoes can be very complex. If the infinitesimal strain ellipse is superposed on the finite ellipse in the most general possible configuration, there are four general fields that result.

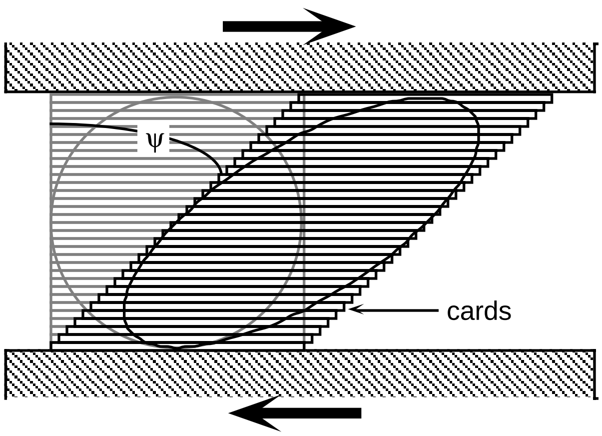


Most general case:

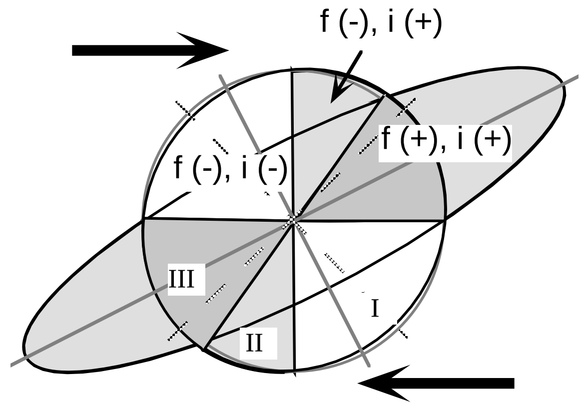
An arbitrary superposition of the infinitesimal ellipse on the finite ellipse. Not very likely in a single progressive deformation

- Field I: lines are shorter than they started, and they will continue to shorten in the next increment;
- Field II: lines are shorter than they started, but will begin to lengthen in the next increment;
- Field III: lines are longer than they started, and will continue to lengthen in the next increment; and
- Field IV: lines are longer than they started, but will shorten in the next increment.

The case for a progressive simple shear is simpler, because one of the lines of no finite extension coincides with one of the lines of no infinitesimal extension. To understand this, think of a card deck experiment.



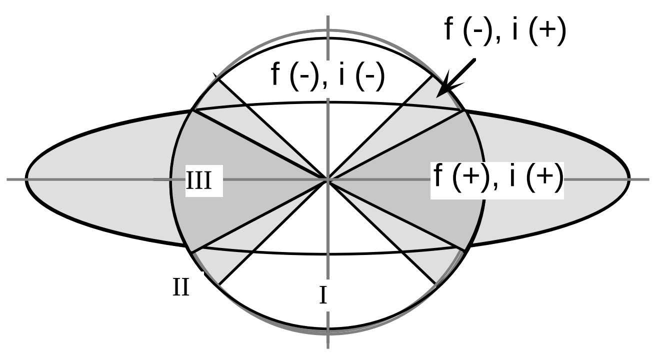
Note that the individual cards never change length or orientation. Thus, they are always parallel to one of the lines of no infinitesimal and no finite extension



Simple Shear

Thus, lines will rotate only in the direction of the shear, and lines that begin to lengthen will never get shorter again during a single, progressive simple shear.

In progressive pure shear, below, you only see the same three fields that exist for simple shear, so, again, lines that begin to lengthen will never get shorter. The difference between pure and simple shear is that, in pure shear, lines within the body will rotate in both directions (clockwise and counterclockwise).

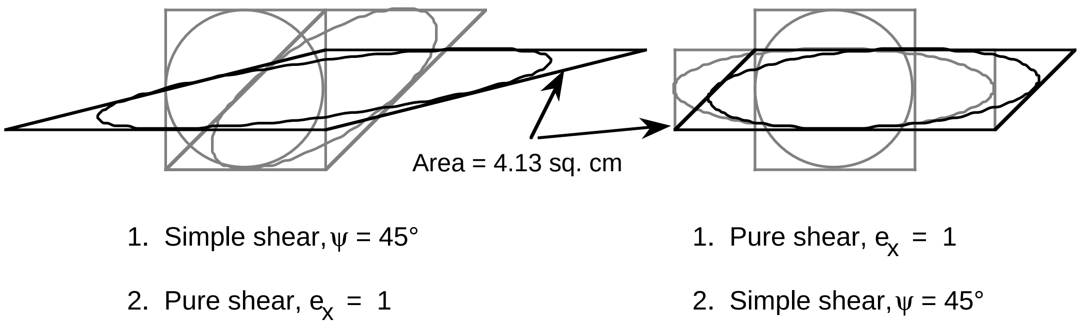


Pure Shear

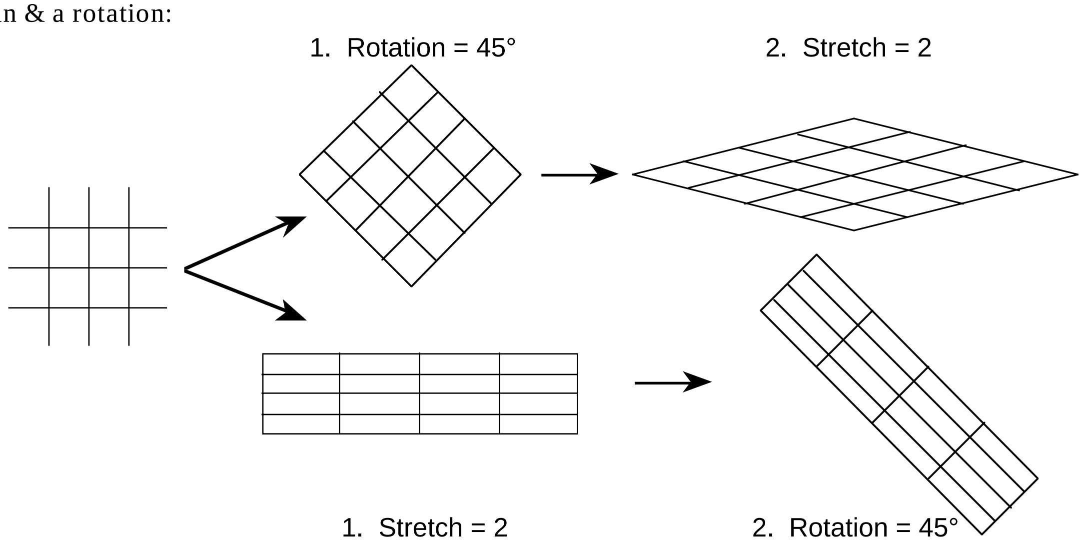
7.4 Superposed Strains & Non-commutability

In general, the order in which strains and rotations of different types are superimposed makes a difference in terms of the final product. This property is called “non-commutability”.

Two strains:

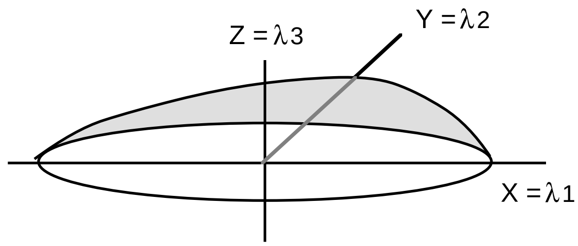


A strain & a rotation:



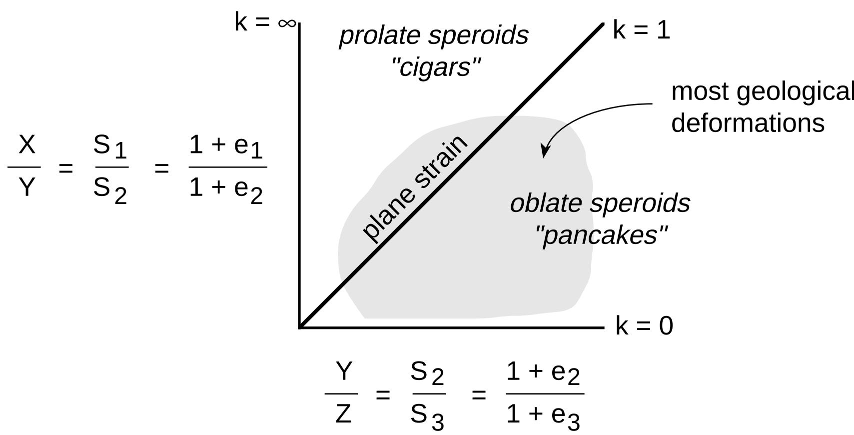
7.5 Plane Strain & 3-D Strain

So far, we’ve been talking about strain in just two dimensions, and implicitly assuming that there’s no change in the third dimension. Strain like this is known as “**plane strain**”. In the most general case, though, strain is three dimensional:



Note that, in three dimensional strain, the lines of no extension become cones of no extension. That is because an ellipsoid intersects a sphere in two cones.

Three-dimensional strains are most conveniently displayed on what is called a Flinn diagram. This diagram basically shows the ratio of the largest and intermediate strain axes, X & Y, plotted against the ratio of the intermediate and the smallest, Y & Z. A line with a slope of 45° separates a field of "cigar"-shaped strain ellipsoids from "pancake"-shaped ellipsoids. All plane strain deformations plot on this line, including, for example, all simple shears.



Lecture 8—Stress I: Introduction

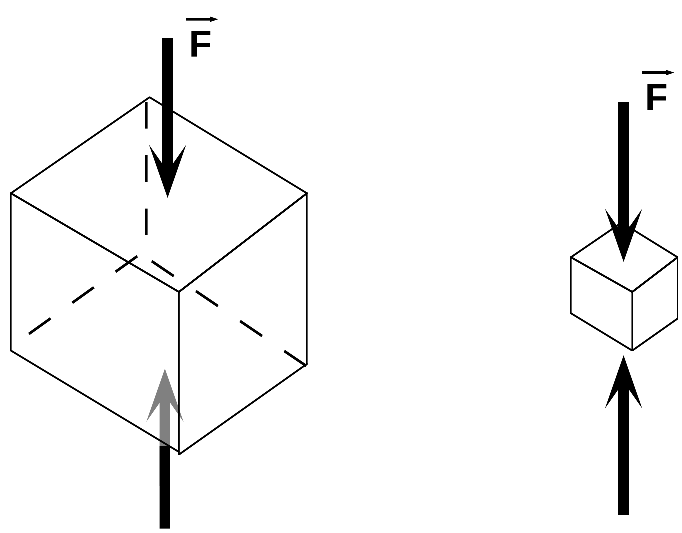
8.1 Force and Stress

I told you in one of the first lectures that we seldom see the forces that are responsible for the deformation that we study in the earth because they are instantaneous, and we generally study old deformations. Furthermore, we cannot measure stress directly. Nonetheless, one of the major goals of structural geology is to understand the distribution of forces in the earth and how those forces act to produce the structures that we see.

There are lots of practical reasons for wanting to do this:

- earthquakes
- oil well blowouts
- what makes the plates move
- why landslides occur, etc.

Consider two blocks of rock. I'm going to apply the same forces to each one:



Your intuition tells you that the smaller block is going to "feel" the force a lot more than the larger block. That's because there are fewer particles in it to distribute the force. Thus, although the two blocks are under the same force, it is more "concentrated" in the little block. To express this, we need to define a new term:

Stress = Force / Area

or as an equation:

$$\vec{\sigma} = \frac{\vec{F}}{A} \tag{8.1}$$

Note that, because force is a vector and area is a scalar, *stress defined in this way must also be a vector.* For that reason, we call it the stress vector or more correctly, a traction vector. When we talk about tractions, it is always with reference to a particular plane.

8.2 Units Of Stress

Stress has units of force divided by area. Force is equal to mass times acceleration. The "official" unit is the Pascal (Pa):

$$\frac{\text{Force}}{\text{Area}} = \frac{\text{mass} \times \text{acceleration}}{\text{Area}} = \frac{\text{kg}\left(\frac{\text{m}}{\text{s}^2}\right)}{\text{m}^2} = \frac{\text{N}}{\text{m}^2} = \text{Pa}$$

In the above equation, N is the abbreviation for "Newton" the unit of force. In the earth, most stresses are substantially bigger than a Pascal, so we more commonly use the unit "megapascal" (Mpa):

$$1 \text{ MPa} = 10^6 \text{ Pa} = 10 \text{ bars} = 9.8692 \text{ atm.}$$

8.3 Sign Conventions:

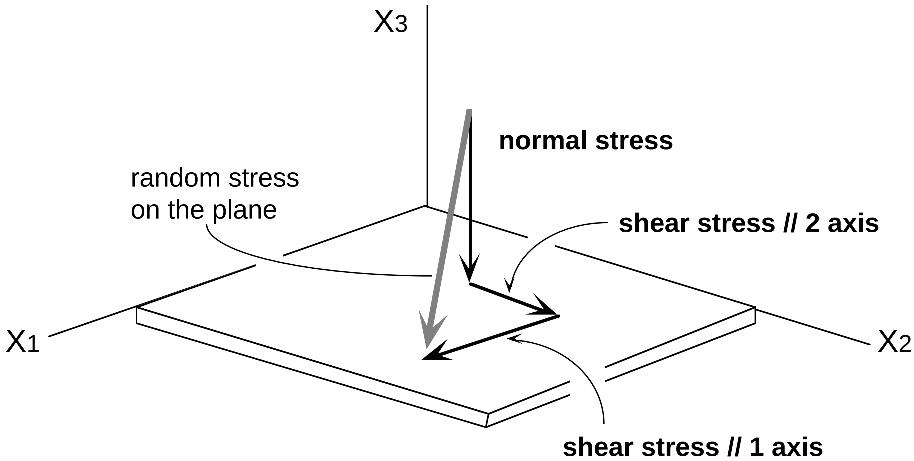
Engineering: compression (-), tension (+)

Geology: compression (+), tension (-)

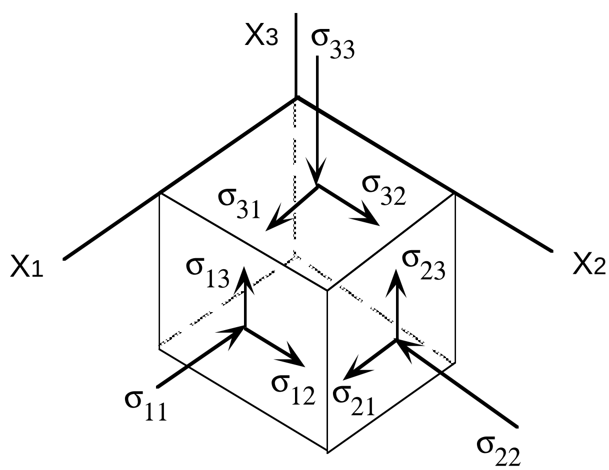
In geology, compression is more common in the earth (because of the high confining pressure). Engineers are much more worried about tensions.

8.4 Stress on a Plane; Stress at a Point

An arbitrary stress on a plane can be resolved into three components:



We can extend this idea to three dimensions to look at stress at a single point, which we'll represent as a very small cube:



In three dimensions, there are nine tractions which define the state of stress at a point. There is a convention for what the subscripts mean:

> the first subscript identifies the plane by indicating the axis which is perpendicular to it

> the second subscript shows which axis the traction vector is parallel to

These nine vectors can be written in matrix form:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \tag{8.2}$$

As you may have guessed, σ_{ij} is the stress tensor. If my cube in the figure, above, is in equilibrium so that it is not rotating, then you can see that

$$\sigma_{12} = \sigma_{21}, \quad \sigma_{13} = \sigma_{31}, \quad \text{and} \quad \sigma_{32} = \sigma_{23}$$

Otherwise, the cube would rotate about one of the axes. Thus, there are only six independent components to the stress tensor. This means that the *stress tensor is a symmetric tensor*.

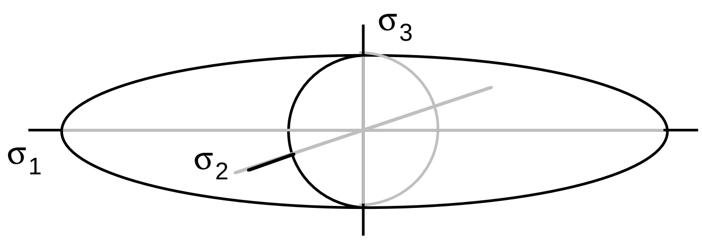
8.5 Principal Stresses

Notice in the "stress on a plane" figure (page 62) that the gray arrow labeled "random stress on a plane" is larger than any of the normal or shear stresses. If we change the orientation of the plane so that it is perpendicular to this arrow then all the shear stresses on the plane go to zero and we are left with only with the gray arrow which is now equal to the normal stresses on the plane. Now let's extend this idea to the block. It turns out that there is one orientation of the block where all the shear stresses on all of the face go to zero and each of the three faces has only a normal stress on it. Then, the matrix which represents the stress tensor reduces to:

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \tag{8.3}$$

In this case the remaining components -- σ_1, σ_2, and σ_3 -- are known as the principal stresses. By convention, σ_1 is the largest and σ_3 is the smallest. People sometimes refer to these as "compression" and "tension", respectively, *but this is wrong*. All three may be tensions or compressions.

You can think of the three principal axes of stress as the major, minor, and intermediate axes of an ellipsoid; this ellipsoid is known as the stress ellipsoid.



8.6 The Stress Tensor

As you may have guessed from the lecture on tensors last time, σ_{ij} is the stress tensor. The stress tensor simply relates the traction vector on a plane to the vector which defines the orientation of the plane [remember, a tensor relates two fields of vectors]. The mathematical relation which describes this relation in general is known as Cauchy's Law:

$$p_i = \sigma_{ij} l_j \tag{8.4}$$

I can use this equation to calculate the stress on any plane in the body if I know the value of the stress tensor in my chosen coordinate system.

8.7 Mean Stress

This is just the average of the three principal stresses. Because the sum of the principal diagonal is just the first invariant of the stress tensor (i.e. it does not depend on the specific coordinate system), you do not have to know what the principal stresses are to calculate the mean stress; it is just the first invariant divided by three:

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} \quad . \tag{8.5}$$

8.8 Deviatoric Stress

With this concept of mean stress, we can break the stress tensor down into two components:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} + \begin{bmatrix} \sigma_{11} - \sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_m \end{bmatrix}. \tag{8.6}$$

The first component is the isotropic part or the mean stress; it is responsible for the type of deformation mechanism as well as dilation. The second component is the deviatoric stress; it is what actually produces the distortion of a body. Note that when you talk about deviatoric stress, the maximum stress is always positive (compressional) and the minimum is always negative (tensional).

8.9 Special States of Stress

- Uniaxial Stress: only one non-zero principal stress, i.e. σ_1 or $\sigma_3 \neq 0$
- Biaxial Stress: one principal stress equals zero, the other two do not
- Triaxial Stress: three non-zero principal stresses, i.e. σ_1, σ_2, and $\sigma_3 \neq 0$
- Axial Stress: two of the three principal stresses are equal, i.e. $\sigma_1 > \sigma_2 = \sigma_3$
- Lithostatic Pressure: The weight of the overlying column of rock:

$$P_{lithostatic} = \int_0^z \rho g dz \approx \rho_{ave} g z$$

- Hydrostatic Pressure: (1) the weight of a column of fluid in the interconnected pore spaces in a rock (Suppe, 1986):

$$P_{fluid} = \rho_{ave} g z_f$$

(2) The mean stress (Hobbs, Means, & Williams, 1976):

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}$$

(3) When all of the principal stresses are equal (Jaeger & Cook, 1976):

$$P = \sigma_1 = \sigma_2 = \sigma_3$$

Although these definitions appear different, they are really all the same. Fluids at rest can support no shear stress (i.e. they offer no resistance to shearing). That is why, by the way, we know that the outer core of the earth is a fluid -- it does not transmit shear waves from earthquakes.

Thus the state of stress is the same throughout the body. This type of stress is also known as Spherical Stress. It is called the spherical stress because it represents a special case in which the stress ellipsoid is a sphere. Thus, every plane in a fluid is perpendicular to a principal stress (because all axes of a circle are the same length) and there is no shear on any plane.

Lecture 9—Vectors & Tensors

Last time, I called stress a tensor; today, I want to give you a glimpse of what that statement actually means. At the same time, we will see a different way of looking at stress (and other tensor properties such as strain) which is very efficient, mathematically. It is much more important that you try to understand the concepts, rather than the specific equations. The math itself, is a part of linear algebra.

We "derived" the stress tensor by considering a small cube whose faces were perpendicular to the axes of an arbitrary coordinate system (arbitrary with respect to the stress on the cube). In other words, we are trying to find something which relates the tractions themselves to the orientations of the planes on which they occur.

9.1 Scalars & Vectors

In your math courses, you have no doubt heard about two different types of quantities:

1. Scalar -- represented by one number. Just a point in space. Some examples:
   - temperature
   - density
   - mass
2. Vector -- represented by three numbers. A line showing direction and magnitude. It only makes sense to talk about a vector with respect to a coordinate system, because of the direction component. Some examples:
   - velocity
   - force
   - displacement

Remember that a vector relates two scalars. For example, the relation between temperature A and B is the temperature gradient which is a vector.

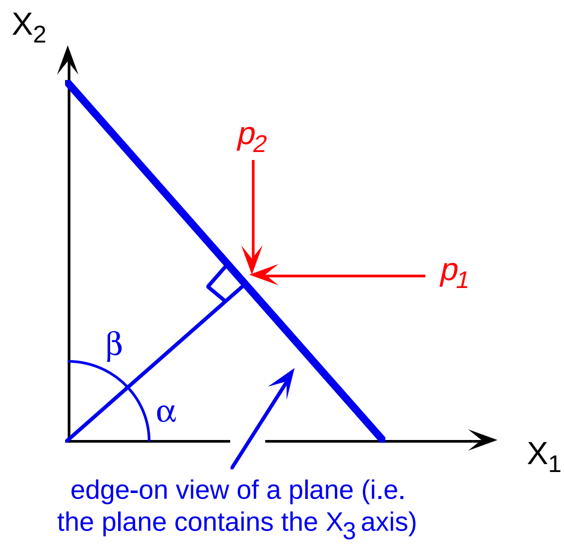
9.2 Tensors

Now we come back to our original question: what type of physical property relates two vectors, or two fields of vectors to each other?

That type of property is called a Tensor:

3. Tensor -- represented by nine numbers. Relates a field of vectors to each other. Generally can be represented as an ellipsoid. Some examples:
    - electrical conductivity
    - thermal conductivity
    - stress
    - strain

The stress tensor relates the orientation of a plane—expressed as the direction cosines of the pole to the plane—to the tractions on that plane. In the diagram, below, if we know the stress tensor, σ_{ij}, then we can calculate the tractions p_1 and p_2 for a plane of any orientation given by α and β:



We can express this relationship by the simple mathematical expression, which is known as **Cauchy's Law**:

$$p_i = \sigma_{ij} l_j . \tag{9.1}$$

9.3 Einstein Summation Convention

The above equation is written in a form that may not be familiar to you because it uses a simple mathematical shorthand notation. We need the shorthand because that equation actually represents a set of three linear equations which are somewhat cumbersome to deal with and write down all the time. There are nine coefficients, ε_{ij}, which correspond to the values of the strain tensor with respect to whatever coordinate system you happen to be using. Those three equations are:

$$\begin{aligned} p_1 &= \sigma_{11} l_1 + \sigma_{12} l_2 + \sigma_{13} l_3 , \\ p_2 &= \sigma_{21} l_1 + \sigma_{22} l_2 + \sigma_{23} l_3 , \\ p_3 &= \sigma_{31} l_1 + \sigma_{32} l_2 + \sigma_{33} l_3 . \end{aligned} \tag{9.2}$$

We could write the same in matrix notation:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} , \tag{9.3}$$

but this is still awkward, so we use the notation above, known as dummy suffix notation, or <u>Einstein Summation Convention</u>. Equations 8-2 can be written more efficiently:

$$p_1 = \sigma_{11} l_1 + \sigma_{12} l_2 + \sigma_{13} l_3 = \sum_{j=1}^{3} \sigma_{1j} l_j ,$$

$$p_2 = \sigma_{21} l_1 + \sigma_{22} l_2 + \sigma_{23} l_3 = \sum_{j=1}^{3} \sigma_{2j} l_j ,$$

$$p_3 = \sigma_{31} l_1 + \sigma_{32} l_2 + \sigma_{33} l_3 = \sum_{j=1}^{3} \sigma_{3j} l_j .$$

From here, it is just a short step to equation 9.1:

$$p_i = \sigma_{ij} l_j , \text{ where i and j both can have values of 1, 2, or 3.}$$

p_1, p_2, and p_3 are the tractions on the plane parallel to the three axes of the coordinate system, X_1, X_2, and X_3, and l_1, l_2, and l_3 are equal to $\cos\alpha$, $\cos\beta$, and $\cos\gamma$, respectively.

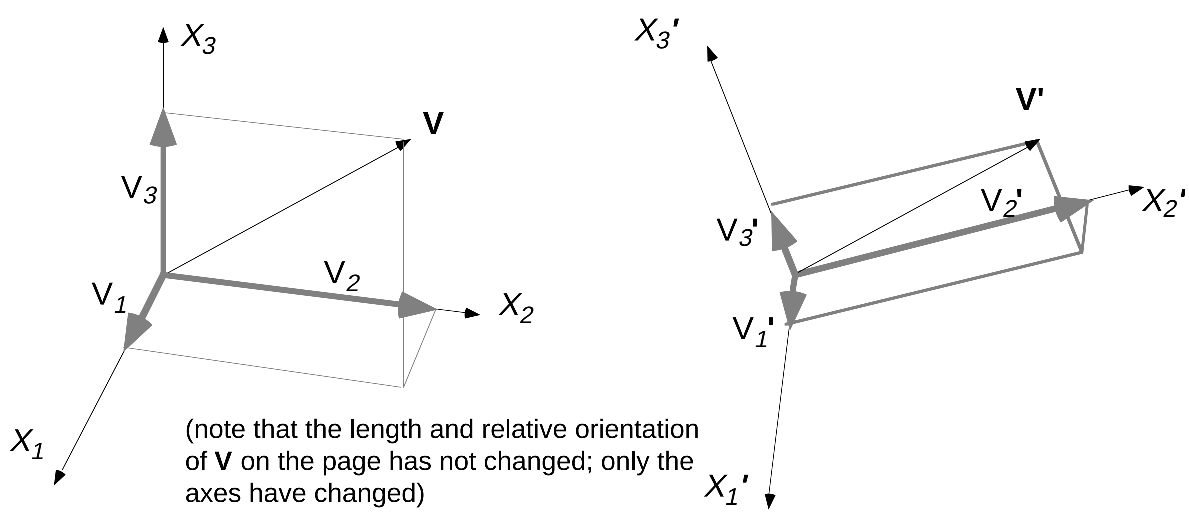
In equation 9.1, because the "j" suffix occurs twice on the right hand side, it is the dummy suffix, and the summation occurs with respect to that suffix. The suffix, "i", on the other hand is the free suffix; it must occur once on each side of each equation.

You can think of the Einstein summation convention in terms of a nested do-loop in any programming language. In a FORTRAN type language, one would write the above equations as follows:

```
Do i = 1 to 3
    p(i) = 0
    Do j = 1 to 3
        p(i) = sigma(i,j)*l(j) + p(i)
    repeat
repeat
```

9.4 Coordinate Systems and Tensor Transformations

The specific values attached to both vectors and tensors -- that is the three numbers that represent a vector or the nine numbers that represent a tensor -- depend on the coordinate system that you choose. The physical property that is represented by the tensor (or vector) is independent of the coordinate system. In other words, I can describe it with any coordinate system I want and the fundamental nature of the thing does not change. As you can see in the diagram, below, for vectors:



(note that the length and relative orientation of **V** on the page has not changed; only the axes have changed)

The same is true of tensors; a strain ellipse has the same dimensions regardless of whether I take a coordinate system parallel to geographic axes or a different one. In the earth, we can use a variety of different coordinate systems; the one most commonly used when we're talking about vectors and tensors is the Cartesian system with direction cosines described earlier:

- north, east, down .

There are times when we want to look at a problem a different way: For example, we are studying a fault and we want to make the axes of the coordinate system parallel to the pole to the fault and the slip direction;

There is a simple way to switch between geographic and fault coordinates: Coordinate transformation, and the related transformations of vectors and tensors.

We're not going to go into the mathematics of transformations (although they are reasonably simple). Just remember that the difference between a tensor and any old random matrix of nine numbers is that you can transform the tensor without changing its fundamental nature.

The nine numbers that represent an infinitesimal strain tensor, or any other tensor, can be represented as a matrix, but not all matrices are tensors. The specific values of the components change when you change the coordinate system, the fundamental nature does not. If I happen to choose my coordinates so that they are parallel to the principal axes of stress, then the form of the tensor looks like:

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

9.5 Symmetric, Asymmetric, & Antisymmetric Tensors

Coming back to our original problem of describing the changes of vectors during deformation, the tensor that relates all those vectors in a circle to their position is known at the displacement gradient tensor.

The displacement gradient tensor, in general, is an asymmetric tensor. What that means is that it has nine independent components, or, if you look at it in matrix form:

$$e_{ij} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}, \quad \text{where} \quad e_{12} \neq e_{21}, \quad e_{13} \neq e_{31}, \quad \text{and} \quad e_{32} \neq e_{23}.$$

If e_{ij} were a symmetric tensor, then $e_{12} = e_{21}$, $e_{13} = e_{31}$, and $e_{32} = e_{23}$, and it would have only 6 independent components.

It turns out that any asymmetric tensor can be broken down into a symmetric tensor and an antisymmetric tensor. So, for the displacement gradient tensor, we can break it down like:

$$e_{ij} = \frac{(e_{ij} + e_{ji})}{2} + \frac{(e_{ij} - e_{ji})}{2}$$

$$= \begin{bmatrix} e_{11} & \frac{(e_{12}+e_{21})}{2} & \frac{(e_{13}+e_{31})}{2} \\ \frac{(e_{21}+e_{12})}{2} & e_{22} & \frac{(e_{23}+e_{32})}{2} \\ \frac{(e_{31}+e_{13})}{2} & \frac{(e_{32}+e_{23})}{2} & e_{33} \end{bmatrix} + \begin{bmatrix} 0 & \frac{(e_{12}-e_{21})}{2} & \frac{(e_{13}-e_{31})}{2} \\ \frac{(e_{21}-e_{12})}{2} & 0 & \frac{(e_{23}-e_{32})}{2} \\ \frac{(e_{31}-e_{13})}{2} & \frac{(e_{32}-e_{23})}{2} & 0 \end{bmatrix}$$

Writing the same equation in a more compact form:

$$e_{ij} = \varepsilon_{ij} + \omega_{ij} \,,$$

where

$$\varepsilon_{ij} = \frac{(e_{ij} + e_{ji})}{2} \qquad \text{and} \qquad \omega_{ij} = \frac{(e_{ij} - e_{ji})}{2} \;.$$

The symmetric part is the infinitesimal strain tensor and the antisymmetric part is the rotation tensor. Written in words, this equation says:

"the displacement gradient tensor = strain tensor + rotation tensor".

Note that the infinitesimal strain tensor is *always* symmetric. Thus, you can think of pure shear as $\omega_{ij} = 0$

and simple shear as $\omega_{ij} \neq 0$.

9.6 Finding the Principal Axes of a Symmetric Tensor

The principal axes of a second order tensor can be found by solving an equation known as the "Characteristic" or "secular" equation. This equation is a cubic, with the following general form:

$$\lambda^3 - \mathrm{I}\lambda^2 - \mathrm{II}\lambda - \mathrm{III} = 0$$

The three solutions for λ are called the eigenvalues; they are the magnitudes of the three principal axes. Knowing those, you can calculate the eigenvectors, which give the orientations of the principal axes. The calculation is generally done numerically using a procedure known as a Jacobi transformation. The coefficients, I, II and III are known as the invariants of the tensor because they have the same values regardless of the orientation of the coordinate system. The first invariant, I, is particularly useful because it is just the sum of the principal diagonal of the tensor. Thus, for the infinitesimal strain tensor, it is always true that:

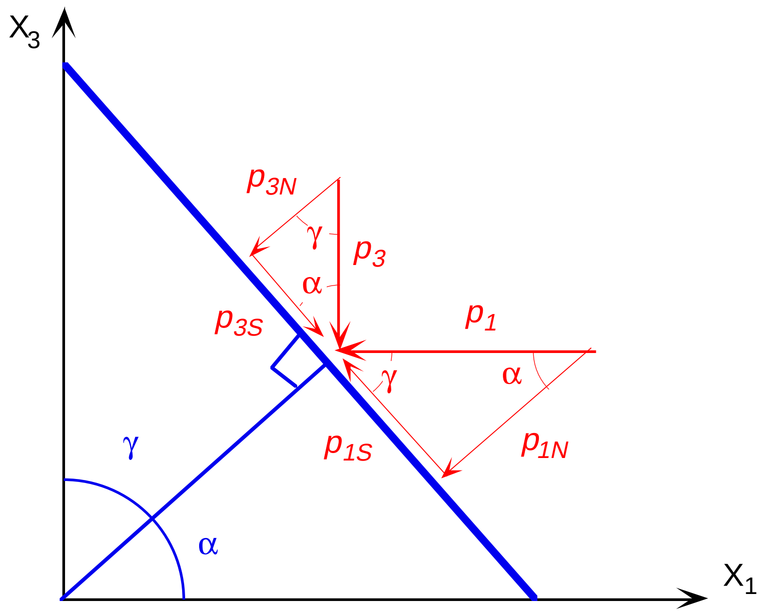
$$\sigma_1 + \sigma_2 + \sigma_3 = \sigma_{11} + \sigma_{22} + \sigma_{33} \ .$$

This is particularly useful when we get to stress and something known as hydrostatic pressure.

LECTURE 10—STRESS II: MOHR'S CIRCLE

10.1 Stresses on a Plane of Any Orientation from Cauchy's law

We would like to be able to calculate the stress on any plane in a body. To do this, we will use Cauchy's Law, which we derived last time.



We will assume that we know the orientations of the principal stresses and that we have chosen our coordinate system so that the axes are parallel to those stresses. This gives us the following matrix for the stress tensor:

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \tag{10.1}$$

The general form of Cauchy's Law is:

$$p_i = \sigma_{ij} l_j \tag{10.2}$$

which, if we expand it out for the case shown above will be:

$$p_1 = \sigma_1 l_1 = \sigma_1 \cos\alpha$$
$$p_3 = \sigma_3 l_3 = \sigma_3 \cos\gamma = \sigma_3 \cos(90-\alpha) = \sigma_3 \sin\alpha$$

If we want to find the normal and shear stresses on the plane, σ_n and σ_s respectively, then we have to decompose the tractions, p_1 and p_3, into their components perpendicular and parallel to the plane. First for p_1:

$$p_{1N} = p_1 \cos\alpha = (\sigma_1 \cos\alpha)\cos\alpha = \sigma_1 \cos^2 \alpha$$
$$p_{1S} = p_1 \sin\alpha = (\sigma_1 \cos\alpha)\sin\alpha$$

and then for p_3:

$$p_{3N} = p_3 \sin\alpha = (\sigma_3 \sin\alpha)\sin\alpha = \sigma_3 \sin^2 \alpha$$
$$p_{3S} = p_3 \cos\alpha = (\sigma_3 \sin\alpha)\cos\alpha$$

Now, the normal stress arrows point in the same direction, so we add them together:

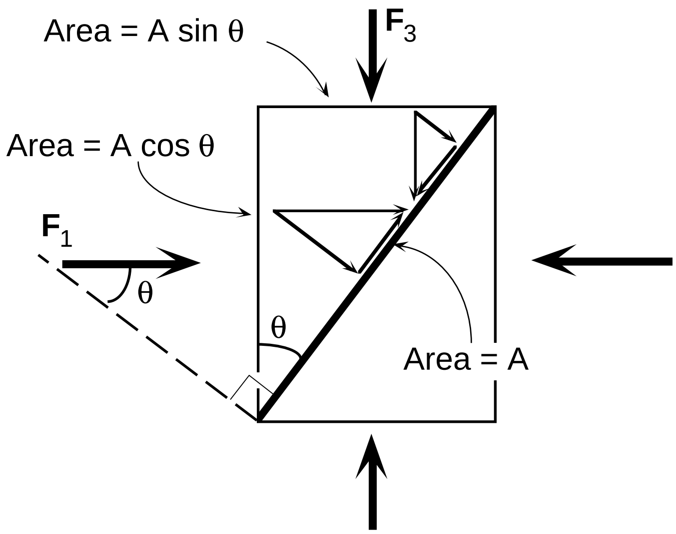
$$\sigma_n = (p_{1N} + p_{3N}) = \sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha \qquad (10.3)$$

The shear stress arrows point in opposite directions so we must subtract them:

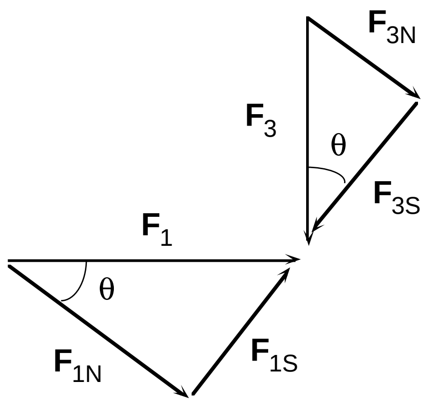
$$\sigma_s = (p_{1S} - p_{3S}) = \sigma_1 \cos\alpha \sin\alpha - \sigma_3 \cos\alpha \sin\alpha = (\sigma_1 - \sigma_3)\cos\alpha \sin\alpha \qquad (10.4)$$

10.2 A more "Traditional" Way to Derive the above Equations

In this section, I will show you a derivation of the same equations which is found in more traditional structural geology text books. The diagram, below, was set up so that there is no shear on the faces of the block. Thus, the principal stresses will be perpendicular to those faces. Also, a very important point to remember in these types of diagrams: *You must always balance forces, not stresses.* So, the basic idea is to balance the forces, find out what the stresses are in terms of the forces, and then write the expressions in terms of the stresses. From the following diagram, you can see that:



10.2.1 Balance of Forces



Force normal to the plane:

$$\mathbf{F}_N = \mathbf{F}_{1N} + \mathbf{F}_{3N}$$

Force parallel to the plane:

$$\mathbf{F}_S = \mathbf{F}_{1S} - \mathbf{F}_{3S}$$

Now, we want to write the normal forces and the parallel (or shear) forces in terms of F_1 and F_3. From simple trigonometry in the above diagram, you can see that:

$$\mathbf{F}_{1N} = \mathbf{F}_1 \cos\theta \ , \quad \mathbf{F}_{1S} = \mathbf{F}_1 \sin\theta$$

and

$$\mathbf{F}_{3N} = \mathbf{F}_3 \sin\theta \ , \quad \mathbf{F}_{3S} = \mathbf{F}_3 \cos\theta.$$

So, substituting these into the force balance equations, we get:

$$\mathbf{F}_N = \mathbf{F}_{1N} + \mathbf{F}_{3N} = \mathbf{F}_1 \cos\theta + \mathbf{F}_3 \sin\theta \tag{10.5}$$

and

$$\mathbf{F}_S = \mathbf{F}_{1S} - \mathbf{F}_{3S} = \mathbf{F}_1 \sin\theta - \mathbf{F}_3 \cos\theta. \quad (10.6)$$

10.2.2 Normal and Shear Stresses on Any Plane

Now that we have the force balance equations written, we just need to calculate what the forces are in terms of the stresses and substitute into the above equations.

$\mathbf{F}_N$ and $\mathbf{F}_S$ act on the inclined plane, which has an area = A. The normal and shear stresses then, are just those forces divided by A:

$$\sigma_n = \frac{\mathbf{F}_n}{A} \quad \text{and} \quad \sigma_s = \frac{\mathbf{F}_s}{A}. \quad (10.7)$$

$\mathbf{F}_1$ and $\mathbf{F}_3$ act on the horizontal and vertical planes, which have different areas as you can see from the first diagram. The principal stresses then, are just those forces divided by the areas of those two sides of the block:

$$\sigma_1 = \frac{\mathbf{F}_1}{A\cos\theta} \quad \text{and} \quad \sigma_3 = \frac{\mathbf{F}_3}{A\sin\theta}. \quad (10.8)$$

Equations 10.7 and 10.8 can be rewritten to give the forces in terms of stresses (a step we skip here) and then we can substitute into the force balance equations, 10.5 and 10.6. For the **normal stresses**:

$$F_N = F_1\cos\theta + F_3\sin\theta = \sigma_n A = \sigma_1 A\cos\theta\cos\theta + \sigma_3 A\sin\theta\sin\theta$$

The A's cancel out and we are left with an expression just in terms of the stresses:

$$\sigma_n = \sigma_1\cos^2\theta + \sigma_3\sin^2\theta \quad (10.9)$$

For the **shear stresses**:

$$F_S = F_1\sin\theta - F_3\cos\theta = \sigma_s A = \sigma_1 A\cos\theta\sin\theta - \sigma_3 A\sin\theta\cos\theta\ .$$

As before, the A's cancel out and we are left with an expression just in terms of the stresses:

$$\sigma_s = \tau = (\sigma_1 - \sigma_3)\sin\theta\cos\theta \quad (10.10)$$

Note that the shear stress is commonly designated by the Greek letter tau, "τ". Also note that we have made an implicit sign convention that clockwise (right-handed) shear is positive. Equations 16.9 and 16.10 are identical to 10.3 and 10.4.

10.3 Mohr's Circle for Stress

Like we did with strain, we can write these equations in a somewhat different form by using the double angle formulas:

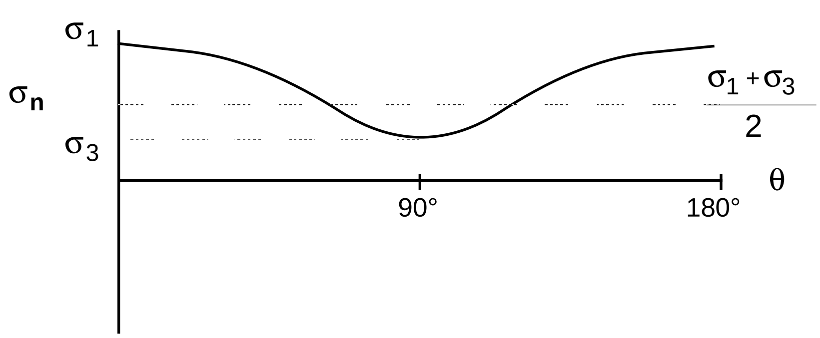
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha .$$

Using these identities, equations 10.9 and 10.10 (or 10.3 and 10.4) become:

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta \tag{10.11}$$

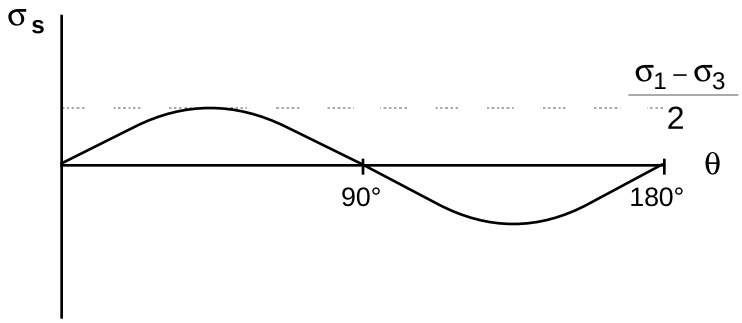
$$\sigma_s = \tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin 2\theta \tag{10.12}$$

The graphs below show how the normal and shear stresses vary as a function of the orientation of the plane, θ:



The above curve shows that:

- maximum normal stress $= \sigma_1$ at $\theta = 0°$
- minimum normal stress $= \sigma_3$ at $\theta = 90°$



This curve shows that:

- shear stress = 0 at $\theta = 0°$ or $90°$

In other words, there is *no shear stress on planes perpendicular to the principal stresses.*

- maximum shear stress = $0.5\,(\sigma_1 - \sigma_3)$ at $\theta = 45°$

Thus, *the maximum shear stress is one half the differential stress*.

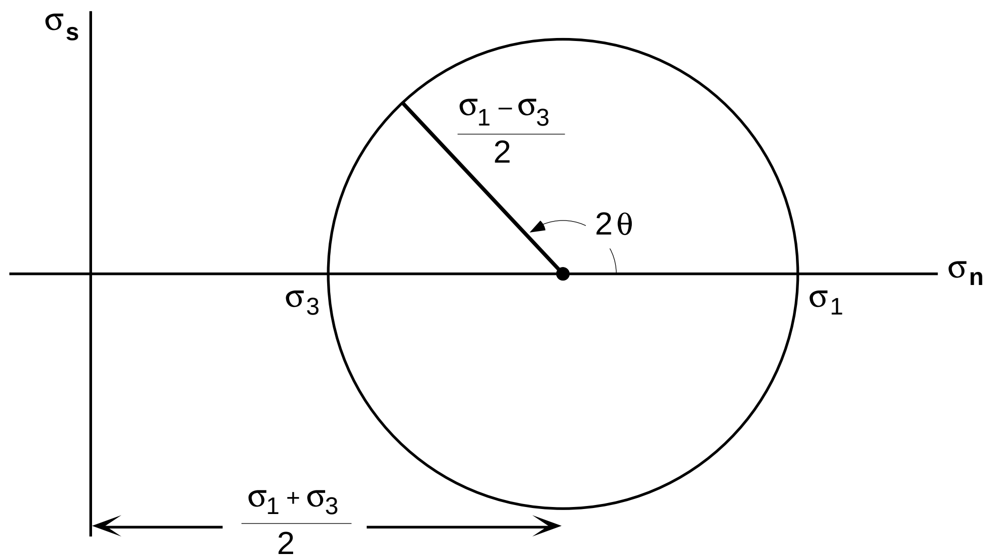
The parametric equations for a circle are:

$$x = c - r\cos\alpha \qquad \text{and} \qquad y = r\sin\alpha\,,$$

so the above equations define a circle with a center on the x-axis and radius:

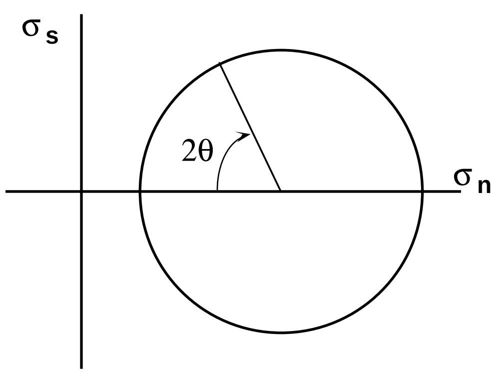
$$(c,\ 0) = \left(\frac{\sigma_1 + \sigma_3}{2},\ 0\right) \quad \text{and} \quad r = \frac{\sigma_1 - \sigma_3}{2}$$

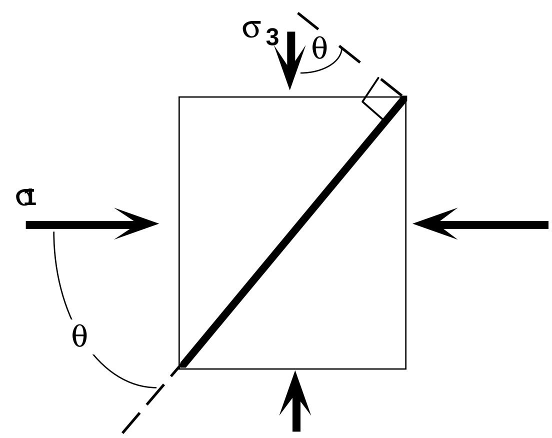
The Mohr's Circle for stress looks like:



10.4 Alternative Way of Plotting Mohr's Circle

Sometimes you'll see Mohr's Circle plotted with the 2θ angle drawn from σ_3 side of the circle:





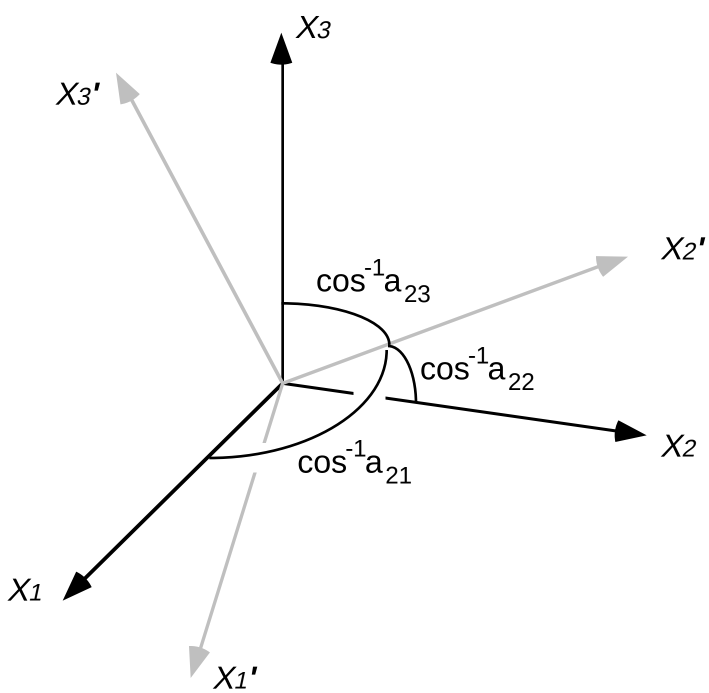
In this case, θ is the angle between the pole to the plane and σ_3, or between the plane itself and σ_1. *It is <u>not</u> the angle between the pole and σ_1.*

10.5 Another Way to Derive Mohr's Circle Using Tensor Transformations

The derivation of Mohr's Circle, above, is what you'll find in most introductory structure textbooks. There is a far more elegant way to derive it using a transformation of coordinate axes and the corresponding tensor transformation. In the discussion that follows, it is much more important to get an intuitive feeling for what's going on than to try and remember or understand the specific equations. This derivation illustrates the general nature of all Mohr's Circle constructions.

10.5.1 Transformation of Axes

This refers to the mathematical relations that relate to orthogonal sets of axes that have the same origin, as shown in the figure, below.



In the diagram, a_{21} is the cosine of the angle between the new axis, X_2', and the old axis, X_1, etc. It is important to remember that, conventionally, the first suffix always refers to the new axis and the second suffix to the old axis. Obviously, there will be three angles for each pair of axes so that there will be nine in all. They are most conveniently remember with a table:

		Old Axes		
		X1	*X2*	*X3*
	X1'	a_{11}	a_{12}	a_{13}
New Axes	*X2'*	a_{21}	a_{22}	a_{23}
	X3'	a_{31}	a_{32}	a_{33}

or, in matrix form:

$$a_{ij} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

Although there are nine direction cosines, they are not all independent. In fact, in the above diagram you can see that, because the third angle is a function of the other two, only two angles are needed to fix one axis and only one other angle -- a total of three -- is needed to completely define the transformation. The specific equations which define the relations between all of the direction cosines are known as the “orthogonality relations.”

10.5.2 Tensor Transformations

If you know the transformation matrix, you can transform any tensor according to the following equations:

$$\sigma'_{ij} = a_{ik} a_{jl} \sigma_{kl} \qquad \text{(new in terms of old)}$$

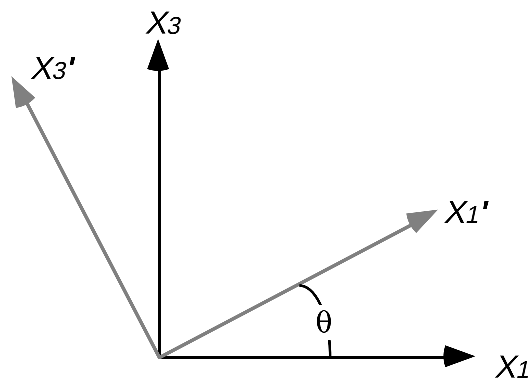
or

$$\sigma_{ij} = a_{ki} a_{lj} \sigma'_{kl} \qquad \text{(old in terms of new).}$$

[These transformations are the key to understanding tensors. The *definition* of a tensor is a physical quantity that describes the relation between two linked vectors. The *test* of a tensor is if it transforms according to the above equations, then it is a tensor.]

10.5.3 Mohr Circle Construction

Any second order tensor can be represented by a Mohr’s Circle construction, which is derived using the above equations simply by making a rotation about one of the principal axes. In the diagram, below, the old axes are parallel to the principal axes of the tensor, σ_{ij}, and the rotation is around the σ_1 axis.



$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

With a rotation of θ about the X_2 axis, the transformation matrix is:

$$a_{ij} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

After a tensor transformation according to the above equations and using the identities $\cos(90 - \theta) = \sin\theta$ and $\cos(90 + \theta) = -\sin\theta$, the new form of the tensor is

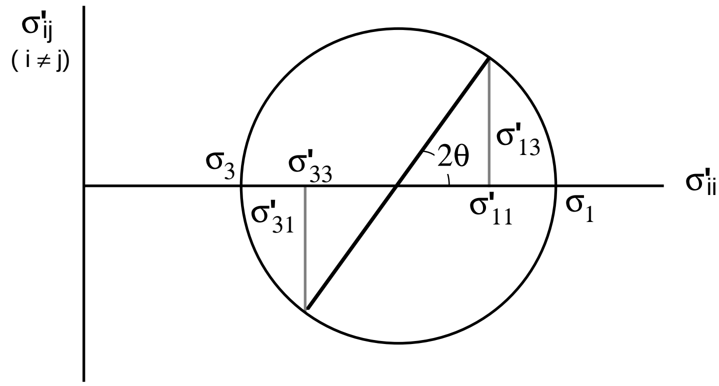
$$\sigma'_{ij} = \begin{bmatrix} \left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right) & 0 & \left(\left(\sigma_3 - \sigma_1\right)\sin\theta\cos\theta\right) \\ 0 & \sigma_2 & 0 \\ \left(\left(\sigma_1 - \sigma_3\right)\sin\theta\cos\theta\right) & 0 & \left(\sigma_1 \sin^2\theta + \sigma_3 \cos^2\theta\right) \end{bmatrix}.$$

Rearranging using the double angle formulas, we get the familiar equations for Mohrs Circle

$$\sigma'_{11} = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta$$

and

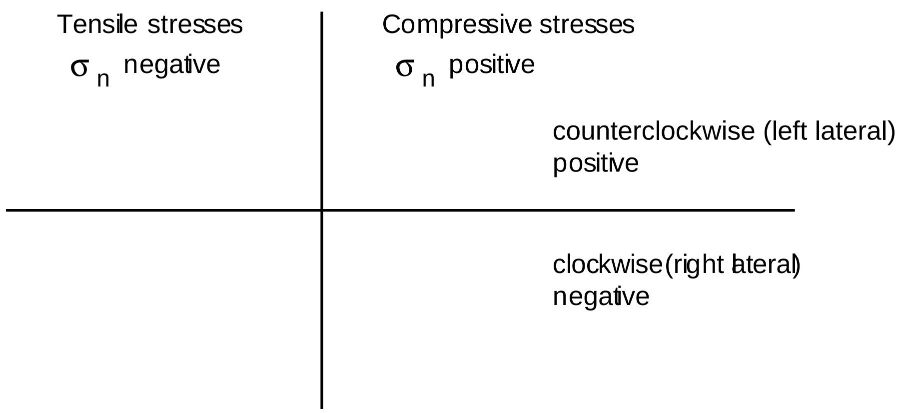
$$\sigma'_{13} = -\left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin 2\theta$$


σ'ij
(i ≠ j)
σ3
σ'33
σ'31
2θ
σ'13
σ'11
σ1
σ'ii

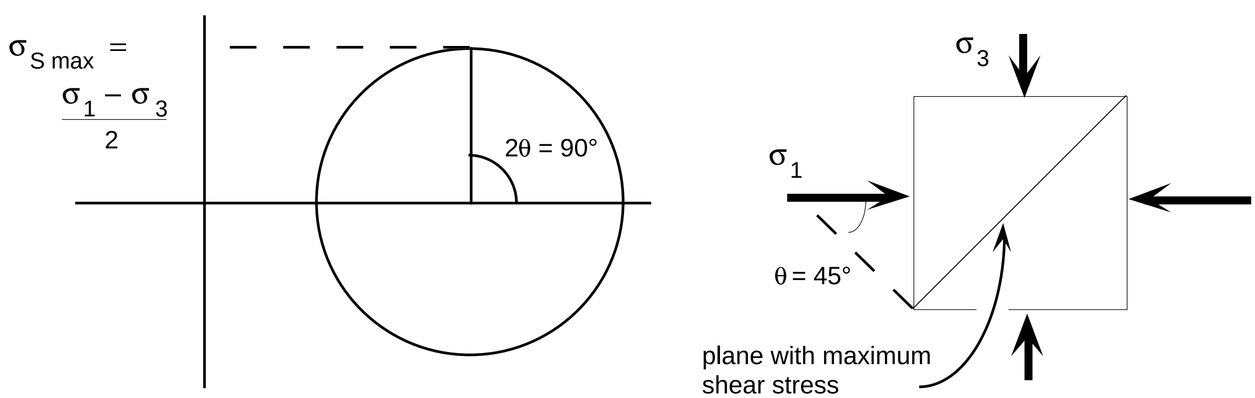
LECTURE 11—STRESS III: STRESS-STRAIN RELATIONS

11.1 More on the Mohr's Circle

Last time, we derived the fundamental equations for Mohr's Circle for stress. We will use Mohr's Circle extensively in this class so it's a good idea to get used to it. The sign conventions we'll use are as follows:

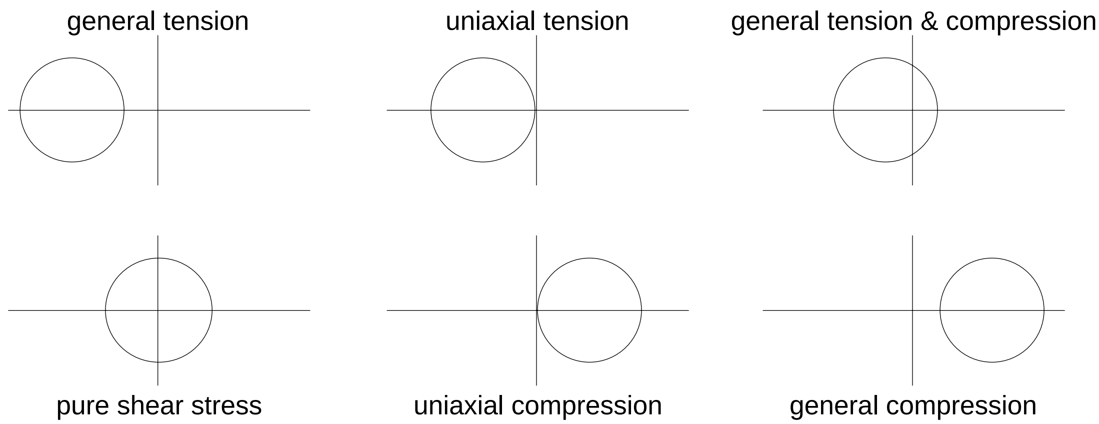


Mohr's circle quickly allows you to see some of the relationships that we graphed out last time:



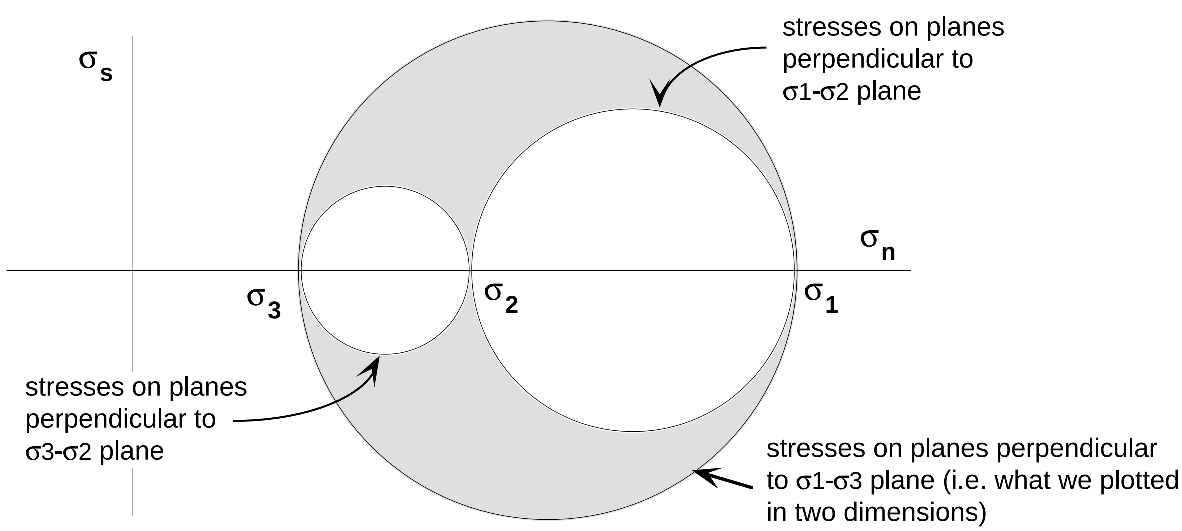
You can see that planes which are oriented at $\theta = 45°$ to the principal stresses ($2\theta = 90°$) experience the maximum shear stress, and that that shear stress is equal to one half the difference of the largest and smallest stress.

The general classes of stress expressed with Mohr's circle are:



11.1.1 Mohr's Circle in Three Dimensions

The concepts that we've been talking about so far are inherently two dimensional [because it is a tensor transformation by rotation about the σ_2 axis]. Even so, the concept of Mohr's Circle can be extended to three dimensions if we consider three separate circles, each parallel to a principal plane of stress (i.e. the plane containing σ_1-σ_2, σ_1-σ_3, or σ_2-σ_3):

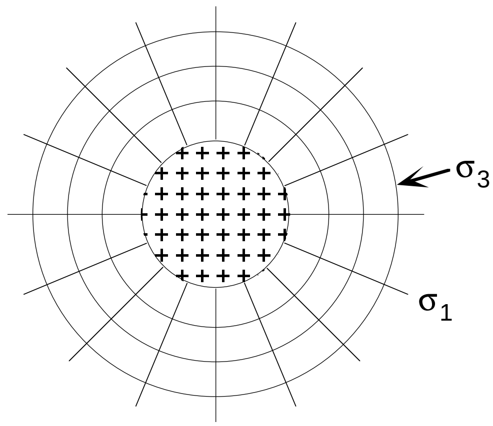


All other possible stresses plot within the shaded area

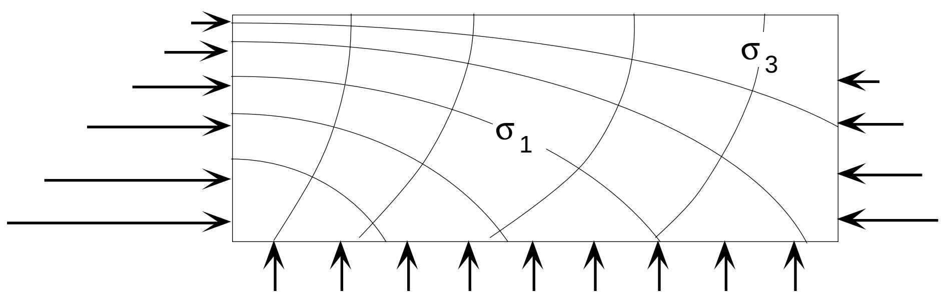
11.2 Stress Fields and Stress Trajectories

Generally within a relatively large geologic body, stress orientation will vary from place to place. This variation constitutes what is known as a stress field.

Stress fields can be portrayed and analyzed using stress trajectory diagrams. In these diagrams, the lines show the continuous variation in orientation of principal stresses. For example, in map view around a circular pluton, one might see the following:



Note that the σ_1 trajectories are always locally perpendicular to the σ_3 trajectories. A more complicated example would be:



this might be an example of a block being pushed over a surface

11.3 Stress-strain Relations

So far, we've treated stress and strain completely separately. But, now we must ask the question of how materials respond to stress, or, what is the relation between stress and strain. The material response to stress is known as **Rheology**.

Natural earth materials are extremely complex in their behavior, but there are some general classes, or models, of material response that we can use. In the most general sense, there are two ways that a material can respond to stress:

1. If the material returns to its initial shape when the stress is removed, then the

deformation is recoverable.

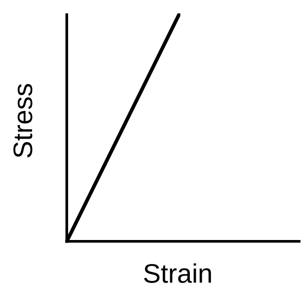
2. If the material remains deformed after the stresses are removed, then the strain is permanent.

11.4 Elasticity

Imagine a body of rock; each time I apply a little more stress, it deforms a bit more:

Stress	Strain
2.5	0.5%
5.0	1.0%
7.5	1.5%
10.0	2.0%
0.0	0.0%

Notice that when I removed the stress in the last increment, the material popped back to its original shape and the strain returned to zero. You can plot data like this on what is known as a stress-strain curve:



The straight line means that there is a constant ratio between stress and strain.

This type of material behavior is known as **elastic**.

Note that part of the definition of elastic behavior is that the material response is instantaneous. As soon as the stress is applied, the material strains by an appropriate amount. When the stress is removed, the material instantly returns to its undeformed state.

11.4.1 The Elasticity Tensor

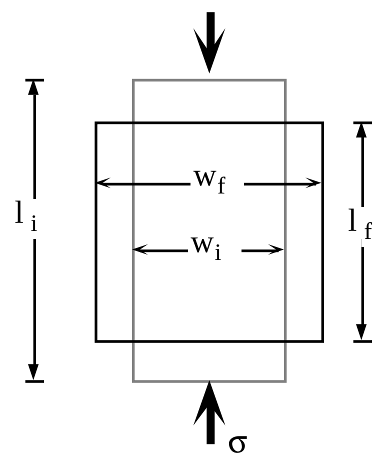
The equation that expresses this linear relation between stress and strain in its most general form is:

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}\,.$$

C_{ijkl} is the elasticity tensor. It is a fourth order tensor which relates two second order tensors. Because all of the subscripts can have values of 1, 2, and 3, the tensor C_{ijkl} has 81 separate components! However, because both the stress and strain tensors are symmetric, the elasticity tensor can have, at most, 36 independent components.

Fortunately, most of the time we make a number of simplifying assumptions and thus end up worrying about four material parameters.

11.4.2 The Common Material Parameters of Elasticity



$$e_l = \frac{l_f - l_i}{l_i}$$

$$e_t = \frac{w_f - w_i}{w_i}$$

With the above measurements, there are several parameters we can derive **Young's Modulus**:

$$E = \frac{\sigma}{e_l} = C_{1111}\ .$$

This is for simple shortening or extensions. For the the ratio of the transverse to longitudinal strain we use **Poissons Ratio**:

$$\upsilon = \frac{e_t}{e_l} = \frac{-E}{C_{1122}}$$

For volume constant deformation (i.e., an incompressible material), $\upsilon = 0.5$, but most rocks vary between 0.25 and 0.33. For simple shear deformations, **Modulus of Rigidity**:

$$\sigma = G e$$

For for uniform dilations or contractions, **Bulk Modulus or Incompressibility**:

$$\sigma = K e$$

All of these parameters are related to each other by some simple equations:

$$G = \frac{E}{2(1+\upsilon)} = \frac{3K(1-2\upsilon)}{2(1+\upsilon)}$$

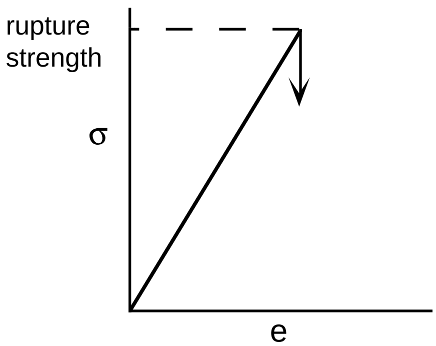
11.5 Deformation Beyond the Elastic Limit

What happens if we keep applying more and more stress to the rock? Intuitively, you know that it can't keep on straining indefinitely. Two things can happen

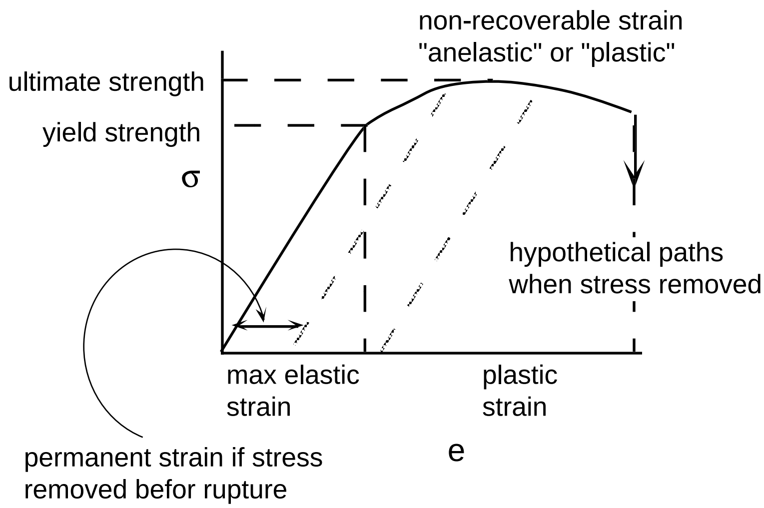
- the sample will break or rupture, or
- the sample will cease deforming elastically and will start to strain faster than the proportional increase in stress.

These two possibilities look like this on stress strain curves:

rupture in elastic realm

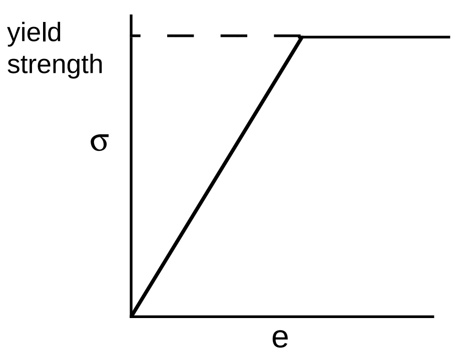


plastic deformation

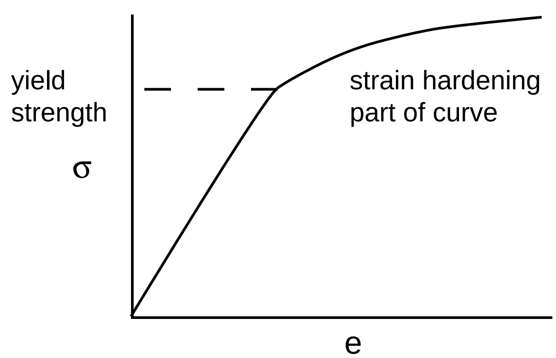


Note that the maximum elastic strains are generally <<5%. There are two forms of plastic deformation:

perfect plastic



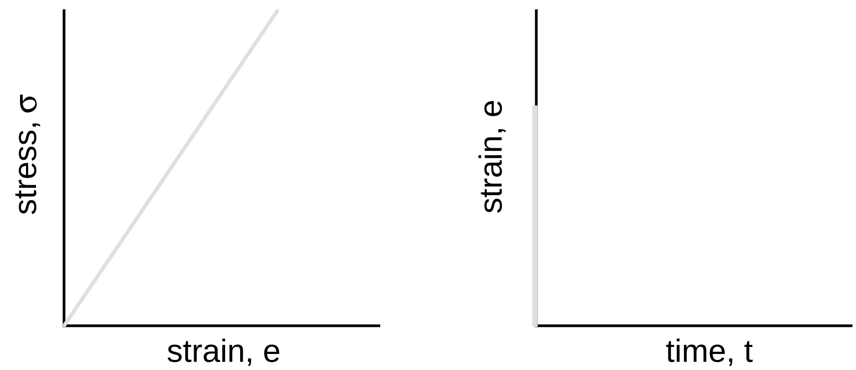
strain hardening



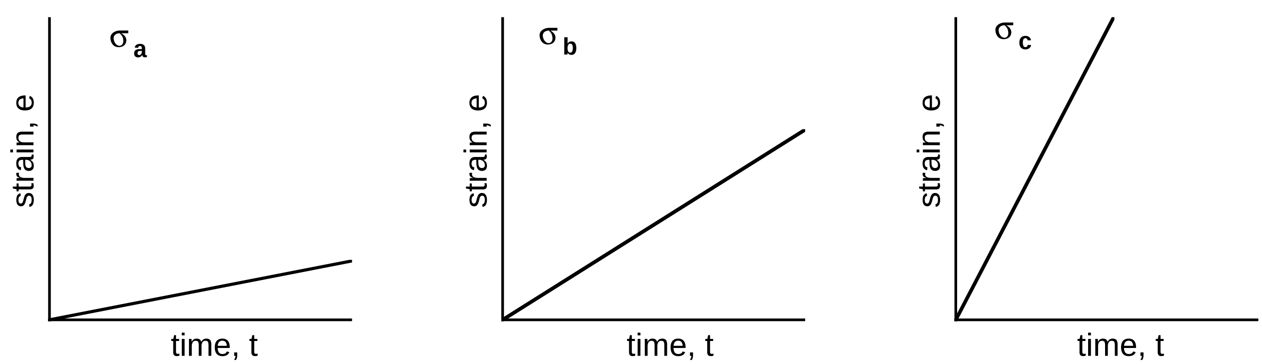
LECTURE 12—PLASTIC & VISCOUS DEFORMATION

12.1 Strain Rate

So far, we haven't really said anything about time except to say the elasticity is instantaneous. You can think of two different graphs:



Time-dependent deformation would have a different response. Suppose I took the same material and did three different experiments on it, each at a different constant stress level:



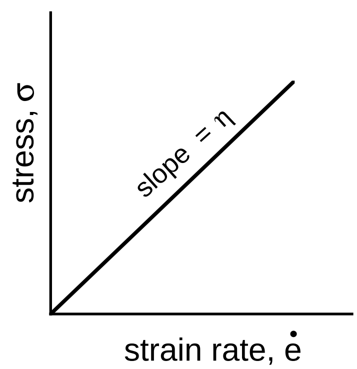
In other words, for different constant stresses, the material deforms at different **strain rates**. In the above graphs, the strain rate is just the slope of the line. Strain rate is the strain divided by time. Because strain has no units, the units of strain rate are inverse time. It is commonly denoted by an "e" with a dot over it: $\dot{e}$. Geological strain rates are generally given in terms of seconds:

$$10^{-16}s^{-1} \le \dot{e}_{geol} \le 10^{-12}s^{-1}.$$

Note that strain rate is <u>not</u> a velocity. Velocity has no reference to an initial shape or dimension and has units of distance divided by time.

12.2 Viscosity

With this idea of strain rate in mind, we can define a new type of material response:



$$\sigma = \eta \dot{e}$$

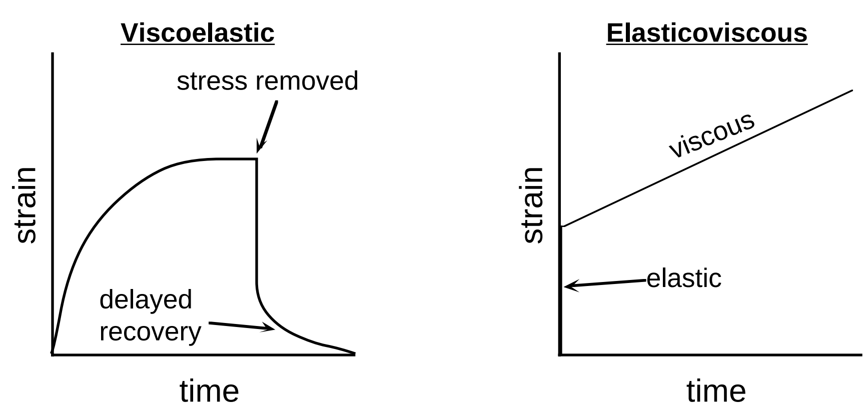
The slope of the curve, η, is the <u>viscosity</u>.

It is a measure of the resistance of the material to flow

A material with a high viscosity flows very slowly. Low viscosity materials flow rapidly. Relative to water, molasses has a high viscosity. When the above curve is straight (i.e. the slope is constant) then we say that it is a **Newtonian fluid**. The important difference between viscous and elastic:

- Viscous -- time dependent
- Elastic -- time independent

Real rocks commonly have a combination of these:



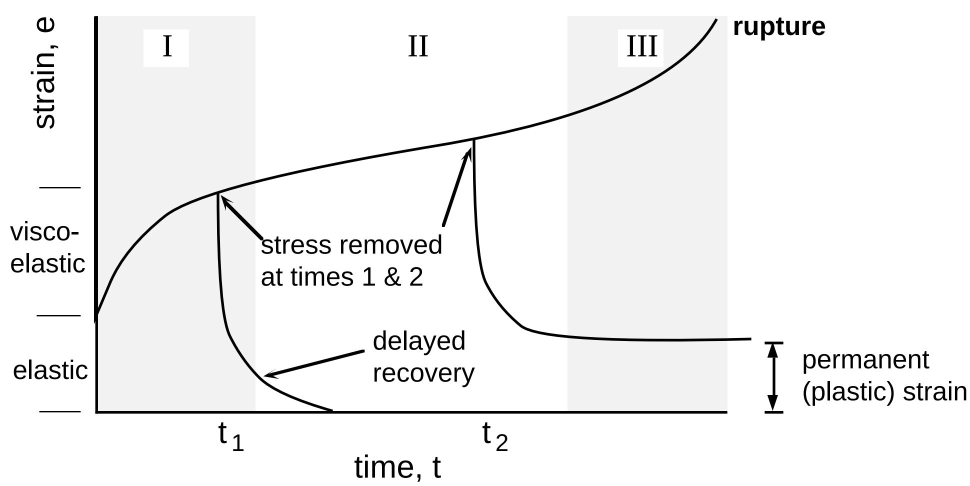
The difference between perfect viscous and perfect plastic:

Perfect viscous -- the material flows under <u>any</u> applied stress

Perfect plastic -- material flows only after a certain threshold stress (i.e. the yield stress) has been reached

<u>12.3 Creep</u>

The viscous material curve on page 93 is idealized. Geological materials deformed under constant stress over long time spans experience several types of rheological behaviors and several strain rates. This type of deformation at constant stress for long times is called **creep**. In general, in long term creep rocks have only 20 - 60% of their total short term strength. As shown in the following diagram, there are three fields:



0 -- Instantaneous elastic strain

I -- Primary or transient creep; strain rate decreases

II -- Secondary or steady state creep; strain rate constant

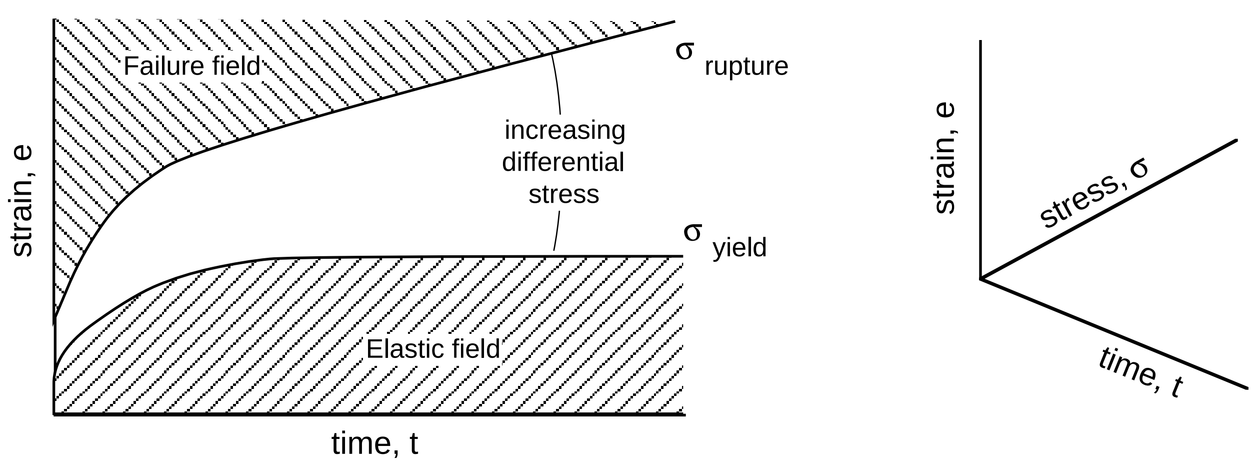
III -- Tertiary or accelerated creep; strain rate goes up

This curve is constructed for constant stress; i.e. stress does not change during the entire length of time. The creep curve has considerable importance for the possibility of predicting earthquakes. Consider some part of the earth's crust under a constant stress for a long period of time. At first the strain is fast (in fact instantaneous) and then begins to slow down until it reaches a steady state. Then, after a long time at steady state, the strain begins to accelerate, *just before rupture, that is the earthquake, occurs.*

12.4 Environmental Factors Affecting Material Response to Stress

There are several factors which change how a material will respond to stress. Virtually all of what we know along these lines comes from experimental work. Usually, when you see stress strain curves for experimental data, the stress plotted is differential stress, $\sigma_1 - \sigma_3$.

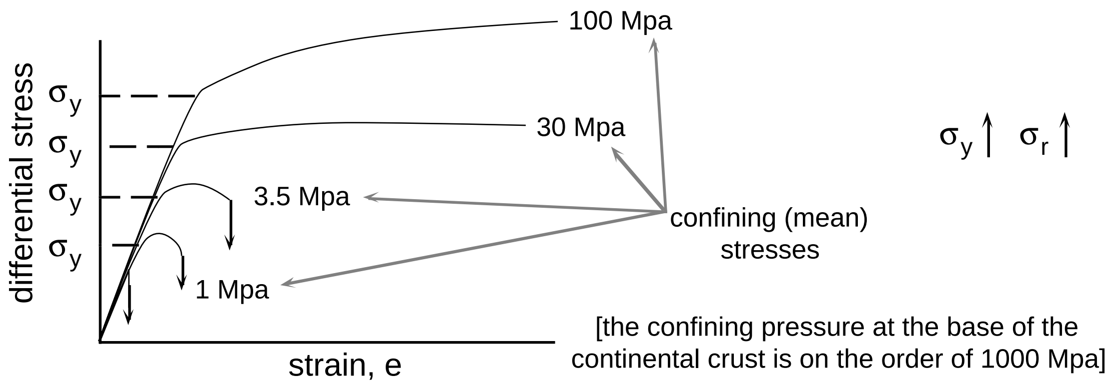
12.4.1 Variation in Stress



As you can see in the above graph, increasing the differential stress drives the style of deformation from elastic to viscous to failure. At low differential stresses, the deformation is entirely elastic or viscoelastic and recoverable. At higher differential stress, the deformation becomes viscous, and finally, at high differential stresses, rupture occurs.

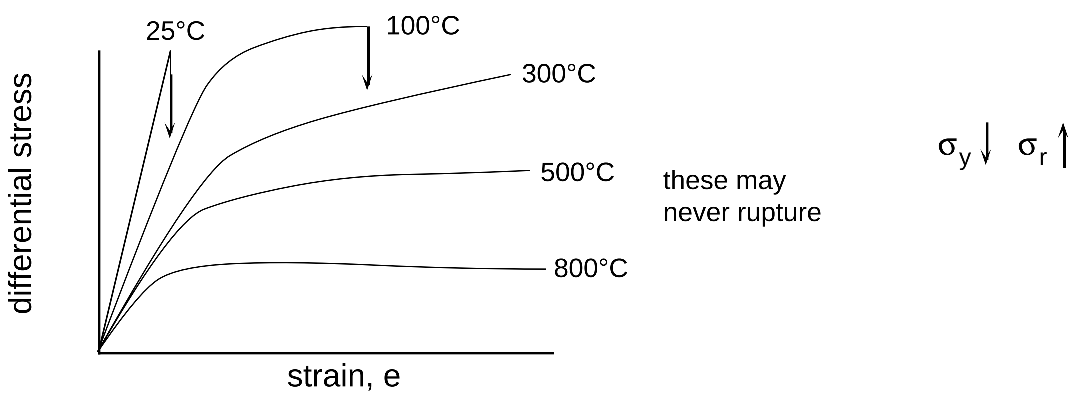
12.4.2 Effect of Confining Pressure (Mean Stress)

An increase in **confining pressure** results in an increase in both the yield stress, σ_y, and the rupture stress, σ_r. The overall effect is to give the rock a greater effective strength. Experimental data shows that:



12.4.3 Effect of Temperature

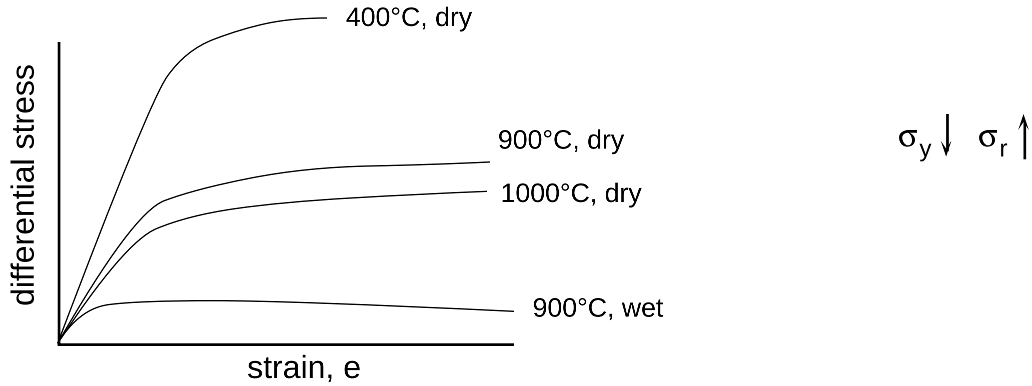
An increase in temperature results in a decrease in the yield stress, σ_y, and an increase in the rupture stress, σ_r. The overall effect is to enlarge the plastic field.



12.4.4 Effect of Fluids

Fluids can have two different effects on the strength of rocks, one at a crystal scale, and one at the scale of the pore space in rocks.

1. Fluids weaken molecular bonds within the crystals, producing an effect similar to temperature; at laboratory strain rates, the addition of water can make a rock 5 to 10 times weaker. With the addition of fluids, the yield stress, σ_y, goes down and the rupture stress, σ_r, goes up:



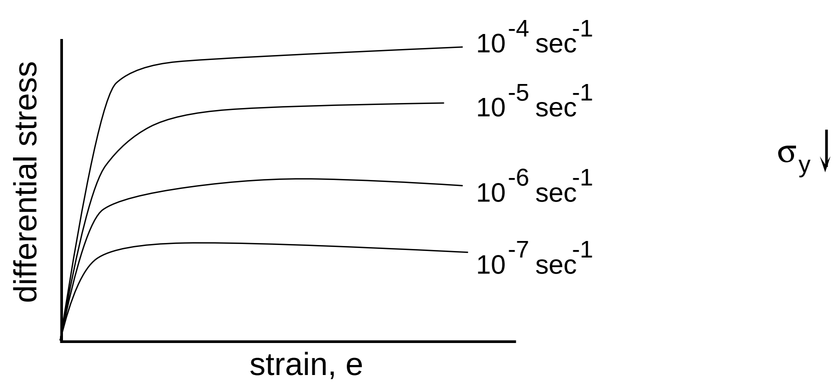
2. If fluid in the pores of the rock is confined and becomes overpressured, it can reduce the confining pressure.

$$P_{effective} = P_{confining} - P_{fluid}$$

As we saw above, a reduced confining pressure tends to reduce the overall strength of the rock.

12.4.5 The Effect of Strain Rate

Decreasing the strain rate results in a reduction of the yield stress, σ_y. In the laboratory, the slowest strain rates are generally in the range of $10^{-6}s^{-1}$ to $10^{-8}s^{-1}$. An “average” geological strain rate of $10^{-14}s^{-1}$ is equivalent to about 10% strain in one million years.

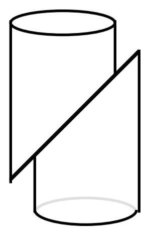


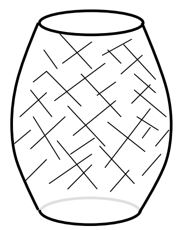
12.5 Brittle, Ductile, Cataclastic, Crystal Plastic

There are several terms which describe how a rock fails under stress. These terms are widely misused in geology. Your will see them again when we talk about fault zones.

Brittle -- if failure occurs during elastic deformation (i.e. the straight line part of the stress-strain curve) and is localized along a single plane, it is called brittle. This is non-continuous deformation, and the piece of rock which is affected by brittle deformations will fall apart into many pieces.

Ductile -- This is used for any rock or material that can undergo large changes in shape (especially stretching) without breaking. Ductile deformation can occur either by cracking and fracturing at the scale of individual grains or flow of individual minerals. In lab experiments, you would see:





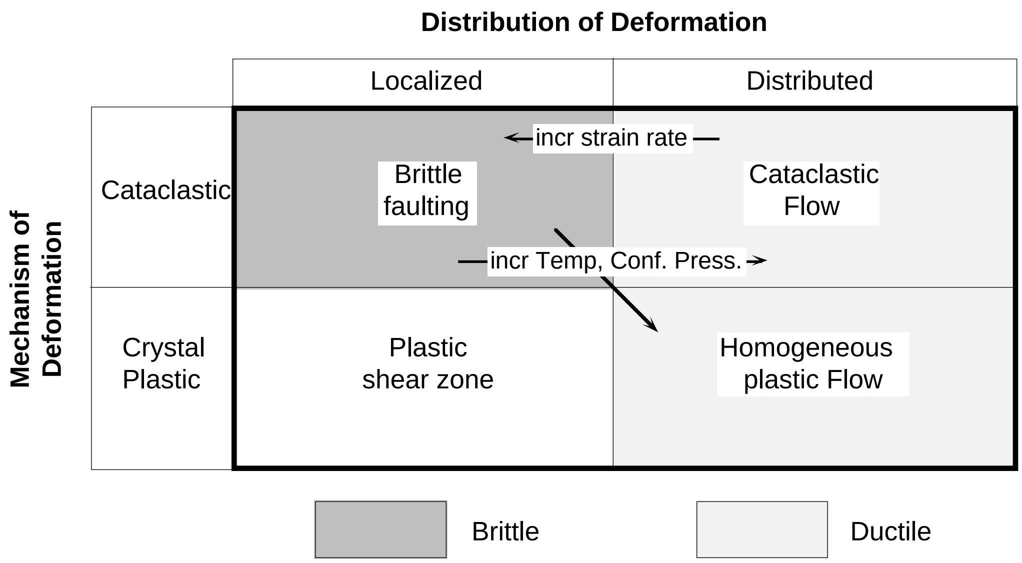
[internal deformation could be by grain-scale fracturing or by plastic flow of minerals; i.e. the deformation mechanism is not specified]

When people talk about the "brittle-ductile" transition, it should be with reference to the above two styles of deformation. Brittle is localized and ductile is distributed. Unfortunately, people usually have a specific deformation mechanism in mind.

Cataclasis (cataclastic deformation) -- Rock deformation produced by fracturing and rotating of individual grains or grain aggregates. This term implies a specific mechanism; both brittle and ductile deformation can be accomplished by cataclastic mechanisms.

Crystal Plastic -- Flow of individual mineral grains without fracturing or breaking. We will talk about the specific types of mechanisms later; for those with some background in material science, however, we are talking in general about dislocation glide and climb and diffusion.

It may help to remember all of these terms with a table (after Rutter, 1986):

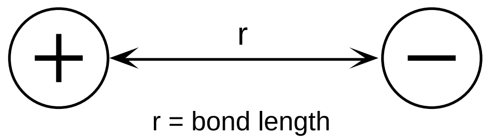

Distribution of Deformation
Localized
Distributed
Mechanism of Deformation
Cataclastic
Crystal Plastic
incr strain rate
Brittle faulting
Cataclastic Flow
incr Temp, Conf. Press.
Plastic shear zone
Homogeneous plastic Flow
Brittle
Ductile

LECTURE 13—DEFORMATION MECHANISMS I: ELASTICITY, COMPACTION

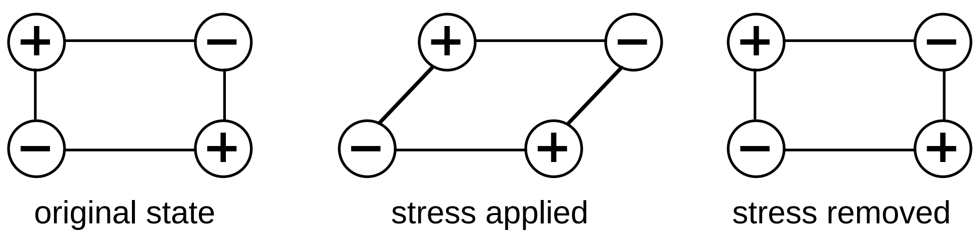
So far, we've been talking just about empirical relations between stress and strain. To further understand the processes we're interested in, we now have to look in more detail to see what happens to a rock on a granular, molecular, and atomic levels.

13.1 Elastic Deformation

If a deformation is recoverable, what does that mean as far as what happens to the rock at an atomic level? It means that no bonds are broken.



In elastic strain, we increase or decrease the bond length, r, but we don't actually break the bond. For example, an elastic simple shear of a crystal might look like:



When the stress is removed, the molecule "snaps" back to its original shape because each bond has a preferred length. What determines the preferred length? It's the length at which the bond has the minimum potential energy. There are two different controls on that potential energy (U):

Potential energy due to attraction between oppositely charge ions

$$U_{attraction} \propto -\frac{1}{r}$$

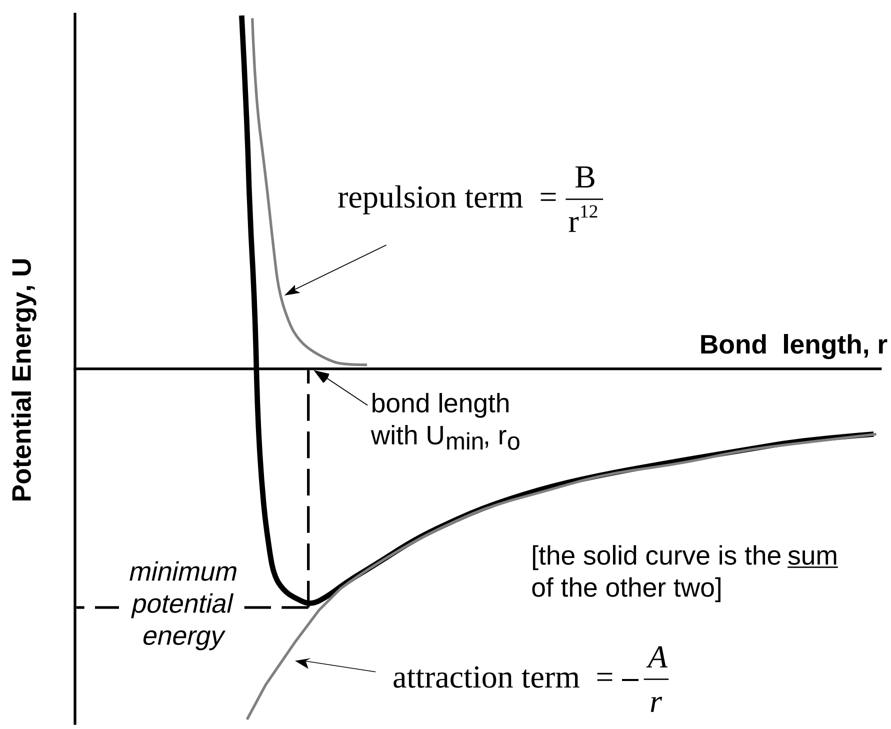
PE due to repulsion from electron cloud overlap

$$U_{repulsion} \propto -\frac{1}{r^{12}}$$

The total potential energy, then, can be written as:

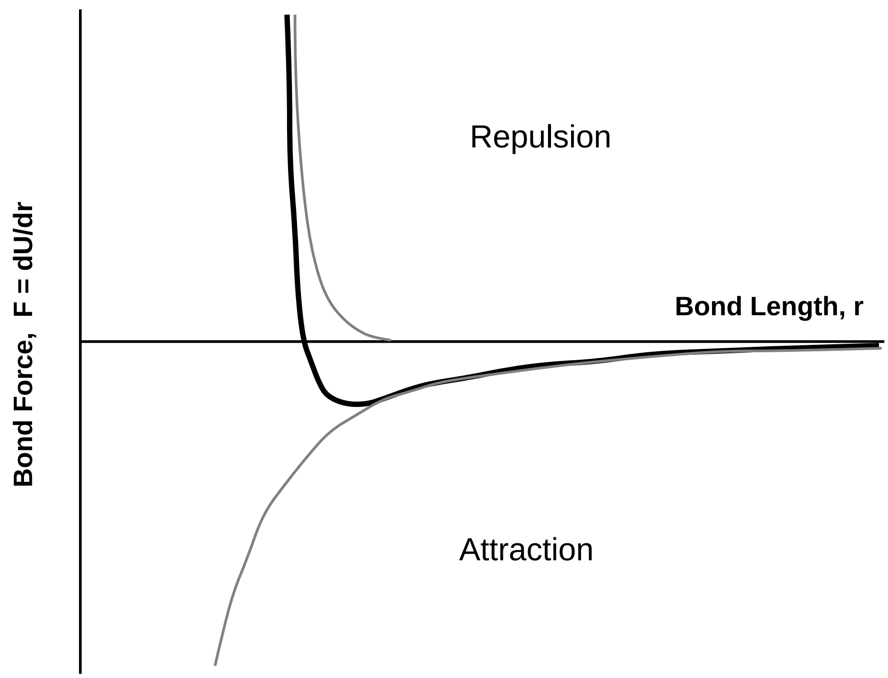
$$U_{total} = \frac{-A}{r} + \frac{B}{r^{12}}$$

where C_1 and C_2 are constants. A graph of this function highlights its important features:



To get the bond force, you have to differentiate the above equation with respect to r:

$$F = \frac{dU}{dr} = \frac{-A}{r^2} + \frac{12B}{r^{13}}$$



Note that repulsion due to electron cloud overlap acts only over very small distances, but it is very strong. The attraction is weaker, but acts over greater distances. These curves show that it is much harder to push the ions together than it is to pull them apart (i.e. the repulsion is stronger than the attraction). At the most basic level, this is the reason for a virtually universal observation:

- rocks are stronger under compression than they are under tension

13.2 Thermal Effects and Elasticity

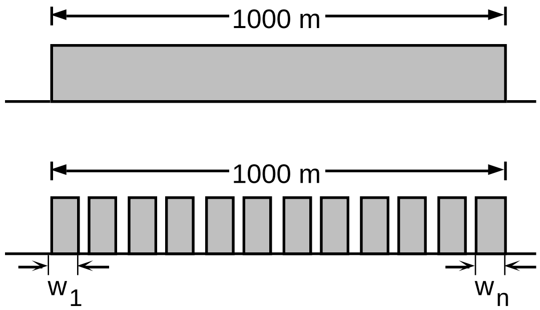
A rise in temperature produces an increase in mean bond length and decrease in potential energy of the bond. This is why rocks have a lower yield stress, σ_y, at higher temperature. The strain due to a temperature change is given by:

$$e_{ij} = \alpha_{ij} \Delta T$$

$\alpha \equiv$ coefficient of thermal expansion

The temperature change, ΔT, is a scalar so the coefficient of thermal expansion, α_{ij}, is a symmetric, second order tensor. It can have, at most, six independent components. The actual number of components depends on crystal symmetry and thus varies between 1 and 6.

A good example of the result of thermal strain are cooling joints in volcanic rocks (e.g. columnar joints in basalts).



Flow erupted at a temperature of 1020°C

Flow cooled to a surface temperature of 20°C

$\Delta T = (T_f - T_i) = -1000°C$

If $\alpha = 2.5 \times 10^{-6}\ °C^{-1}$ and $\Delta T = -1000°C$, then the strain on cooling to surface temperature will be

$$e = \alpha \Delta T = 2.5 \times 10^{-6}\ °C^{-1} \times -1000°C = -2.5 \times 10^{-3}.$$

If the initial length of the flow is 1000 m, then the change in length will be:

$$e = \frac{w_f - w_i}{w_i} = \frac{\Delta w}{w_i} \qquad \Rightarrow \qquad \Delta w = e\, w_i = -2.5 \times 10^{-3} (1000\ m) = -2.5\ m.$$

The joints form because the flow shrinks by 2.5 m. Because the flow is welded to its base, it cannot shrink uniformly but must pull itself apart into columns. If you added up all the space between the columns (i.e. the space occupied by the joints) in a 1000 m long basalt flow, it would total 2.5 m:

$$1000\ m - \Sigma\, w_n = 2.5\ m\ .$$

13.3 Compaction

Compaction is a process that produces a permanent, volumetric strain. It involves no strain of individual grains or molecules within the grains; it is the result of the reduction of pore space between the grains.

Porosity is defined as:

$$\phi = \frac{V_p}{V_p + V_s} = \frac{\text{volume of the pores}}{\text{total volume}},$$

and the **void ratio** as:

$$\theta_v = \frac{V_p}{V_s} = \frac{\text{volume of the pores}}{\text{volume of the solid}}$$

Much compaction occurs in a sedimentary basin during diagenesis and is not tectonic in origin.

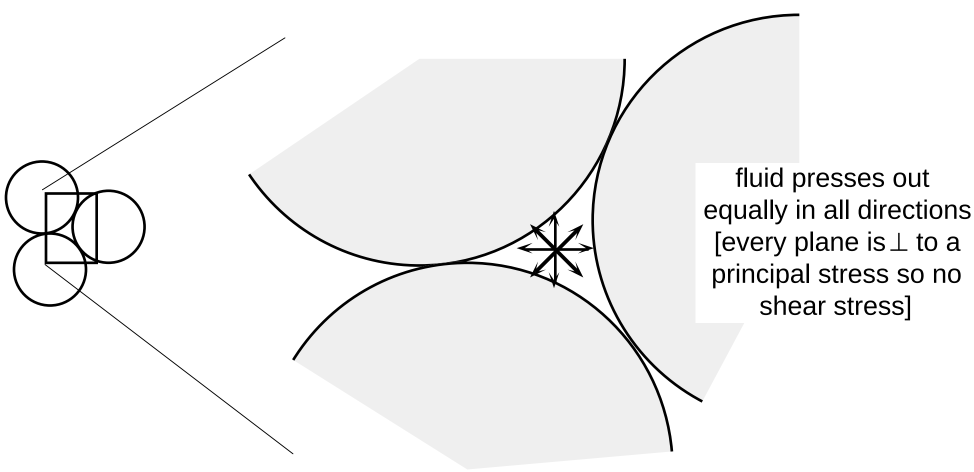
There is an empirical relationship between compaction and depth in a sedimentary basin known as Athy's Law:

$$\phi = \phi_o e^{-az}$$

where z = the depth, a = some constant, and ϕ_o is the initial porosity ["e" means exponential not strain].

13.4 Role of Fluid Pressure

Compaction is usually considered hand in hand with fluid pressure. This is just the pressure of the fluids which fill the pores of the rock. Usually, the fluid is water but it can also be oil, gas, or a brine. We shall see in the coming days that fluid pressure is very important for the overall strength of the rock.



13.4.1 Effective Stress

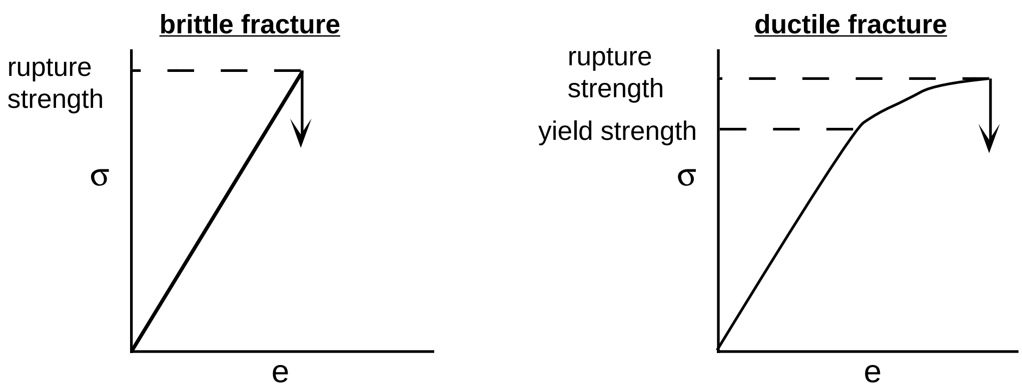
The role of fluids in a rock is to reduce the normal stress across the grain to grain contacts in the rock without changing the shear stresses. We can now define a new concept, the effective stress which originally comes from Terzaghi in soil mechanics, but appears equally applicable to rocks.

$$\sigma^*_{ij} = \begin{bmatrix} (\sigma_{11} - P_f) & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & (\sigma_{22} - P_f) & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & (\sigma_{33} - P_f) \end{bmatrix}$$

Note that only the principal diagonal (i.e. the normal stresses) of the matrix is affected by the pore pressure.

LECTURE 14—DEFORMATION MECHANISMS II: FRACTURE

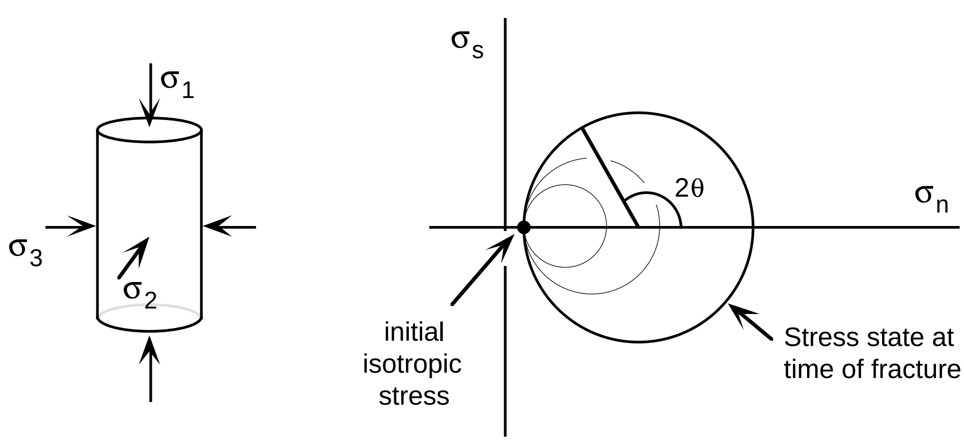
A very important deformation mechanism in the upper part of the Earth's crust is known as **fracture**. Fracture just means the breaking up into pieces. There are two basic types as shown in our now familiar stress-strain curves:



In brittle fracture, there is no permanent deformation before the rock breaks; in ductile fracture, some permanent deformation does occur before it breaks. Fracture is strongly dependent on confining pressure and the presence of fluids, but is not as strongly dependent on temperature.

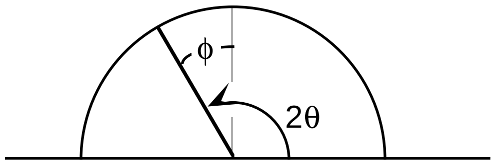
14.1 The Failure Envelop

The Mohr's circle for stress is a particularly convenient way to look at fracture. Suppose we do an experiment on a rock. We will start out with an isotropic stress state (i.e. $\sigma_1 = \sigma_2 = \sigma_3$) and then gradually increase the axial stress, σ_1, while holding the other two constant:



If we look in detail at the configuration of the Mohr's Circle when fracture occurs, there is something very curious:

[detail of previous figure]

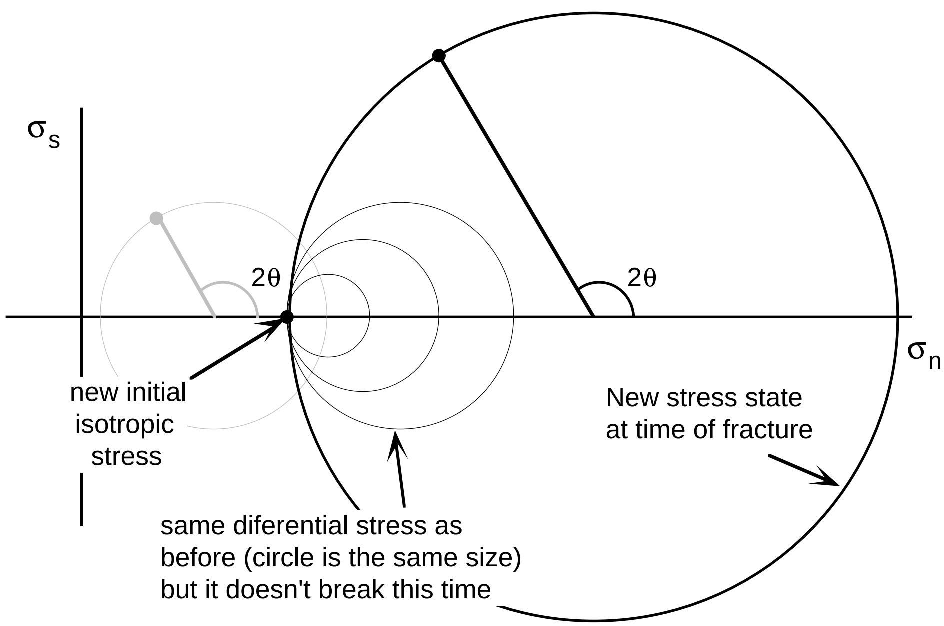


ϕ = angle of internal friction

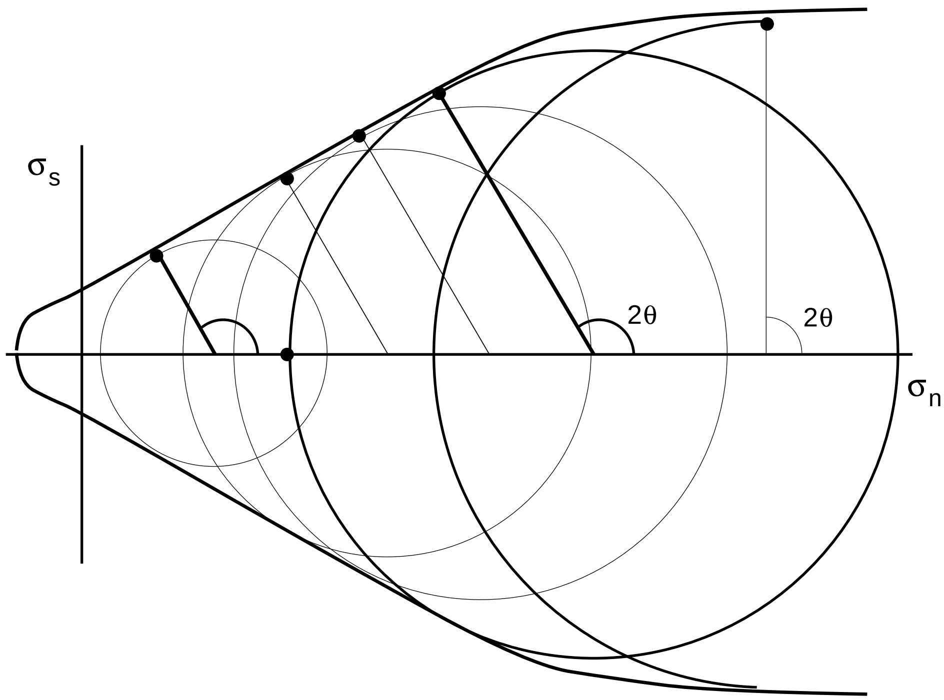
$2\theta = 90° + \phi$

The fracture does <u>*not*</u> *occur on the plane with the maximum shear stress* (i.e. $2\theta = 90°$). Instead, the angle, 2θ, is greater than 90°. The difference between 2θ at which the fracture forms and 90° is known as the **angle of internal friction** and is usually given by the Greek letter, ϕ.

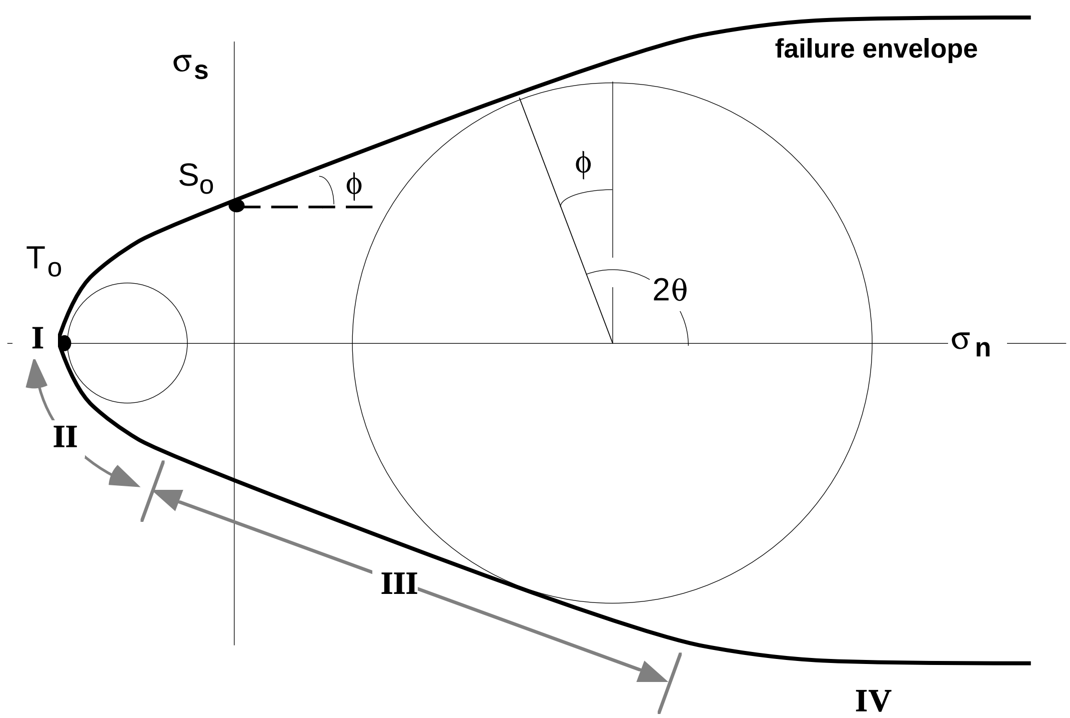
Now lets do the experiment again at a higher confining pressure:



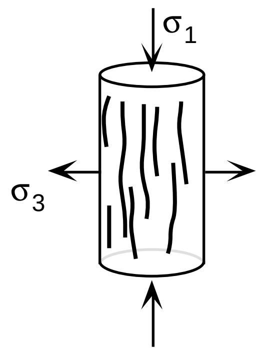
In fact, we can do this sort of experiment at a whole range of different confining pressures and each time there would be a point at which the sample failed. We can construct an "envelop" which links the stress conditions on each plane at failure. Stress states in the rock with Mohr's circles smaller than this envelop would not result in failure; any stress state in which the Mohr's Circle touch or exceeded the envelop would produce a fracture of the rock:



In general, we see a **failure envelop** which has four recognizable parts to it:

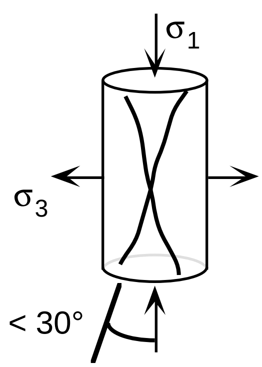


Field I -- Tensile fracture: You can see that the Mohr's circle touches the failure envelop in *only* one place. The 2θ angle is 180°; thus, the fractures form parallel to σ_1 and perpendicular to σ_3. The point T_o is known as the Tensile strength. Note that, because the Mohr's circle intersects the failure envelope at a principal stress, there is no shear stress on the planes in this case. The result is that you make joints instead of faults.



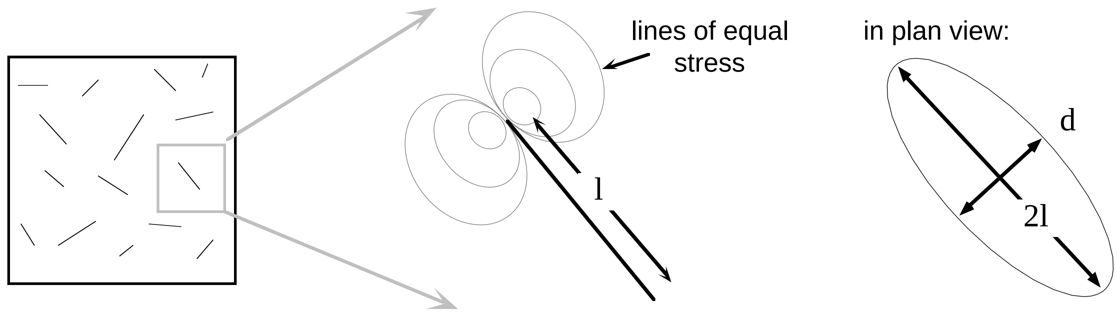
$-30 \leq To \leq -4$ Mpa

Field II -- Transitional tensile behavior: this occurs at $\sigma_1 \approx |3T_o|$. The circle touches the envelop in two places, and, $120° \leq 2\theta \leq 180°$:



$30° \leq \phi \leq 90°$

The shape of the trans-tensile part of the failure envelop is determined by cracks in the material. These cracks are known as **Griffith Cracks** after the person who hypothesized their existence in 1920. Cracks are extremely effective at concentrating and magnifying stresses:



The tensile stress at the tip of a crack is given by:

$$\sigma \approx \frac{2}{3}\sigma_3 \frac{(2l)^2}{d}$$

The sizes of cracks in rocks are proportional to the grain size. Thus, fine-grained rocks will have shorter cracks and be stronger under tension than coarse-grained rocks. The equations for the trans-tensile part of the failure envelope, predicted by the Griffith theory of failure are:

$$\sigma_s^2 - 4T_o\ \sigma_n - 4T_o^2 = 0$$

or

$$\sigma_s = 2\sqrt{T_o\left(\sigma_n + T_o\right)}$$

Field III -- Coulomb behavior: This portion of the failure envelop is linear, which means that there is a linear increase in strength with confining pressure. This is very important because it is characteristic of the behavior of the majority of rocks in the upper crust of the earth. The equation for this part of the failure envelop is:

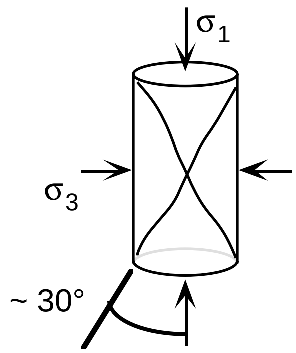
$$\sigma_s = s_o + \sigma_n \tan\phi = s_o + \sigma_n\,\mu$$

In the above equation:

μ = coefficient of internal friction

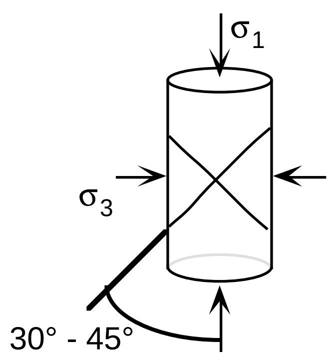
and

s_o = the cohesion



$$\phi \approx 30°$$
$$2\theta \approx 120°$$
$$\theta \approx 60°$$

Field IV -- Ductile failure (Von Mises criterion): This occurs at high confining pressure and increasing temperature. Here the fracture planes become nearer and nearer to the planes of maximum shear stress, which are located at 45°. There is a constant differential stress at yield.



$$0° \leq \phi \leq 30°$$
$$90° \leq 2\theta \leq 120°$$
$$45° \leq \theta \leq 60°$$

14.2 Effect of Pore Pressure

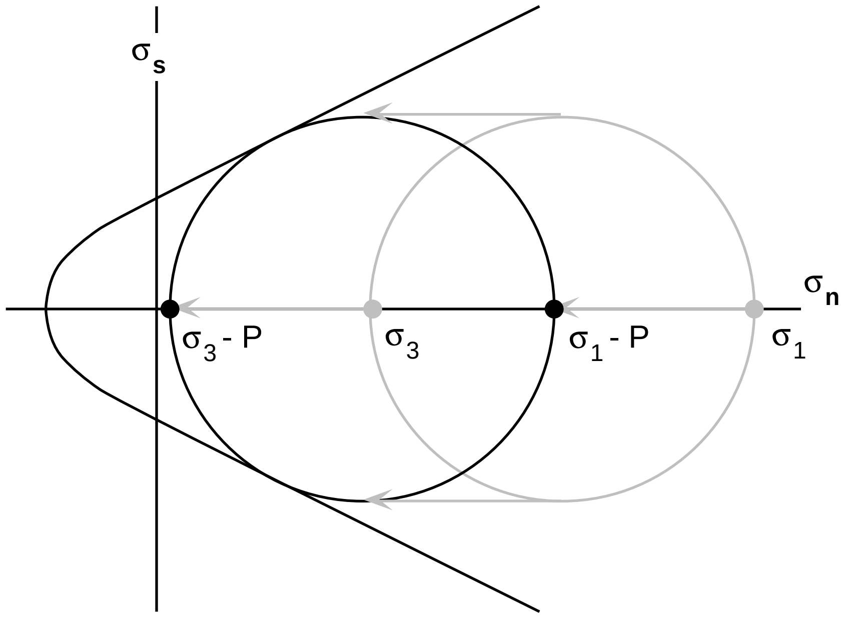
Last time, we saw that the pore fluid pressure counteracts, or reduces, the normal stress but not the shear stress:

$$\text{Effective stress} = \sigma^*_{ij} = \begin{bmatrix} (\sigma_{11} - P_f) & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & (\sigma_{22} - P_f) & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & (\sigma_{33} - P_f) \end{bmatrix}$$

Taking this into account, the equation for Coulomb fracture then becomes:

$$\sigma_s = s_o + (\sigma_n - P_f) \tan\phi = s_o + \sigma_n{}^* \mu$$

The result is particularly striking on a Mohr's Circle:



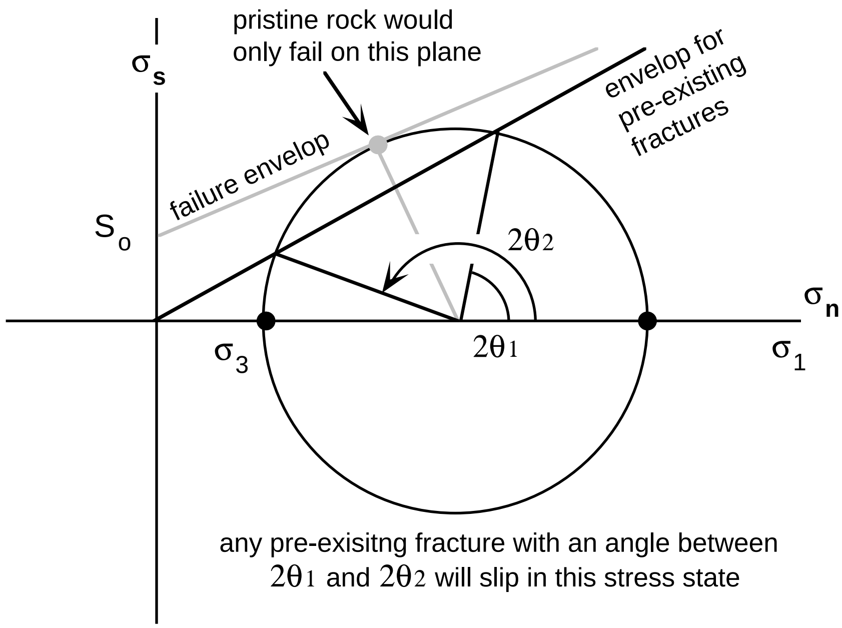
Because the pore fluid pressure changes the effective normal stress but does not affect the shear stress, the radius of the Mohr's Circle stays the same but the circle shifts to the left. A high enough pore fluid pressure may drive the circle to the left until it hits the failure envelop and the rock breaks. Thus, pore pressure weakens rocks.

This effect is used in a practical situation when one wants to increase the permeability and porosity of rocks (e.g. in oil wells to help petroleum move through the rocks more easily, etc.). The process is known as **hydrofracturing** or **hydraulic fracturing**. Fluids are pumped down the well and into the surrounding rock until the pore pressure causes the rocks to break up.

14.3 Effect of Pre-existing Fractures

Rock in the field or virtually anywhere in the upper part of the Earth's crust have numerous preexisting fractures (e.g. look at the rocks in the gorges around Ithaca). These fractures will affect how the rock subsequently fails when subjected to stress. Two things occur:

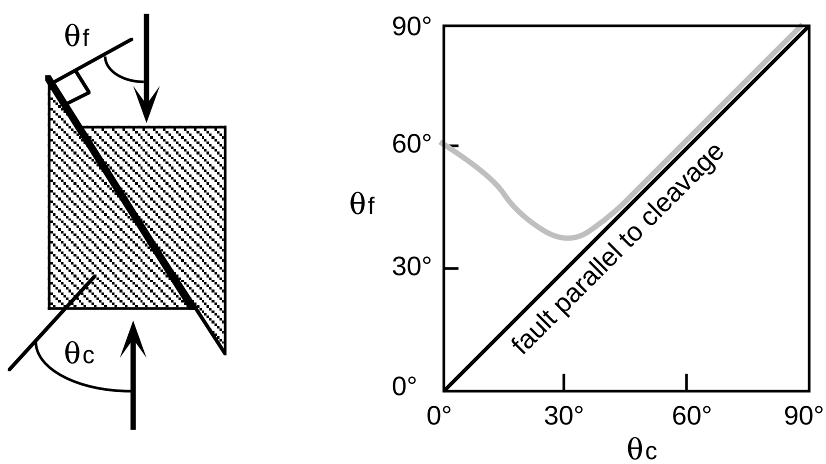
- S_o, the cohesion, goes virtually to zero
- μ, the coefficient of friction changes to a coefficient of sliding friction



The equation for the failure envelop for preexisting fractures is

$$\sigma_s = \sigma_n * \mu_f$$

This control by preexisting features can be extended to metamorphic foliations.



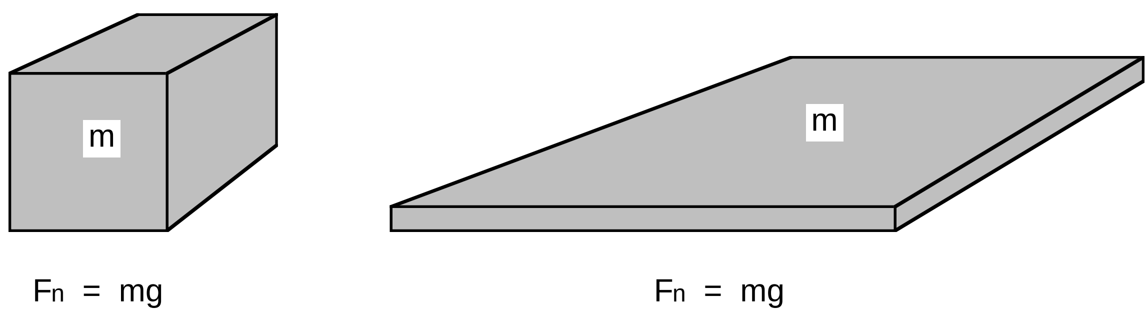
14.4 Friction

The importance of friction was first recognized by Amontons, a French physicist, in 1699. Amontons presented to the French Royal Academy of Science two laws, the second of which was very controversial:

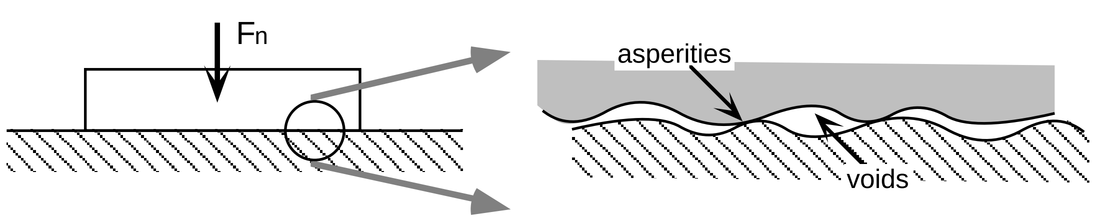
- Amontons First Law -- Frictional resisting force is proportional to the normal force

- Amontons Second Law -- Frictional resisting force is independent of the area of surface contact

The second law says in effect that you can change the surface area however you want but, if the normal force remains the same, the friction will be the same. You have to be intellectually careful here. The temptation is to think about increasing the surface area with the implicit assumption that the mass of the object will change also. But if that happens, then the normal force will change, violating the first law. So, when you change the surface area, you must also change the mass/area.



Much latter, Bowden provided an explanation for Amontons' second law. He recognized that the microscopic surfaces are very much rougher than it appears from our perspective. [Example: if you shrunk the Earth down to the size of a billiard ball, it would be smoother than the ball.] Thus its surface area is very different than the macroscopic surface area:



At the points of contact, or **asperities**, there is a high stress concentration due to the normal stress.

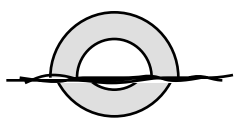
LECTURE 15—DEFORMATION MECHANISMS III: PRESSURE SOLUTION & CRYSTAL PLASTICITY

15.1 Pressure Solution

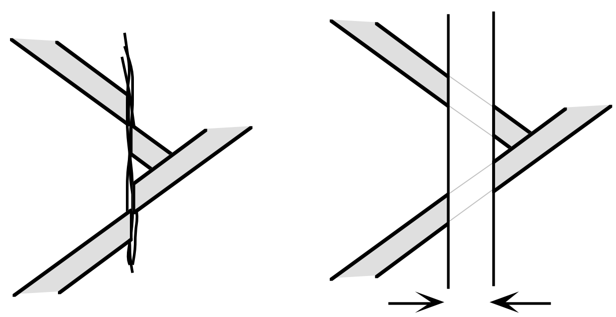
15.1.1 Observational Aspects

One of the very common deformation mechanisms in the upper crust involves the solution and re-precipitation of various mineral phases. This process is generally, and loosely, called pressure solution.

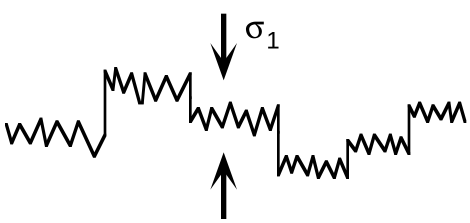
Evidence that pressure solution has occurred in rocks:



crinoid stem or other fossil



material removed by pressure solution

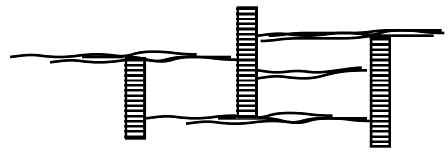


Stylolites

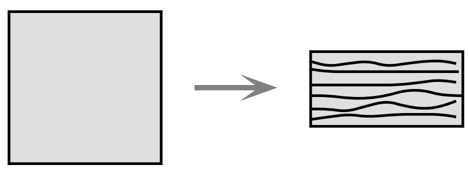
Classic morphology: jagged teeth with concentrations of insoluable residue. This is common in marbles (e.g. particularly well seen in polished marble walls). Many stylolites don't have this form.

Although we commonly think of stylolites as forming in limestones and marbles, they are also very common in siliceous rocks such as shale and sandstone.

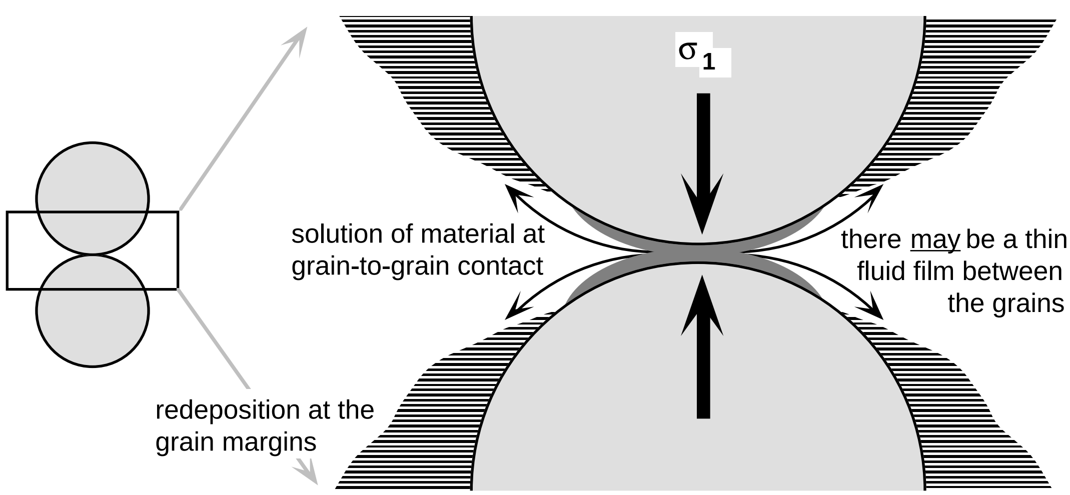
Sometimes, we see veins and stylolites nearby, indicating that volume is preserved on the scale of the hand sample or outcrop. In this case, the veins are observed to be approximately perpendicular to the stylolites:



More commonly, there is much more evidence for removal of material than for the local re-precipitation. Then, there is a net volume decrease; you see shortening but no extension. The rocks in the Delaware Water Gap area, for example, have experienced more than 50% volume loss due to pressure solution.



What actually happens to produce pressure solution? No one really knows, but the favored model is that, because of the high stress concentration at grain contacts, material there is more soluble. Material dissolved from there migrates along the grain boundary to places on the sides of the grains, where the stress concentration is lower, and is deposited there. This model is sometimes called by the name "fluid assisted grain boundary diffusion" because the material diffuses along a thin fluid film at the boundary of the grain:



This process is probably relatively common during diagenesis.

Not all pressure solution can be called a diffusional process because, as we will see later, diffusion acts slowly and over short distances. In the case where there is a net volume reduction at hand sample or outcrop scale, there has to be has to be large scale flushing of the material in solution out of the system by long distance migration of the pore fluids.

15.1.2 Environmental constrains on Pressure Solution

Temperature -- most common between ~50° and 400°C. Thus, you will see it best developed in rocks that are between diagenesis and low grade metamorphism (i.e., greenschist facies).

Grain Size -- at constant stress, pressure solution occurs faster at smaller grain sizes. This is because grain surface area *increases* with decreasing grain size.

Impurities/clay -- the presence of impurities such as clay, etc., enhances pressure solution. It may be that the impurities provide fluid pathways.

The switch from pressure solution to mechanisms dominated by crystal plasticity is controlled by all of these factors. For two common minerals, the switch occurs as follows:

	Upper Temperature Limit for Pressure Solution	
Grain Size	**Quartz**	**Calcite**
100 µm	450°C	300°C
1000 µm	300°C	200°C

These temperatures are somewhere in lower greenschist facies of metamorphism.

15.2 Mechanisms of Crystal Plasticity

Many years ago, after scientists had learned a fair amount about atom structure and bonding forces, they calculated the theoretical strengths of various materials. However, the strengths that they predicted turned out to be up to *five orders of magnitude* higher than what they actually observed in laboratory experiments. Thus, they hypothesized that crystals couldn't be perfect, but must have **defects** in them. We now know that there are three important types of crystal defects:

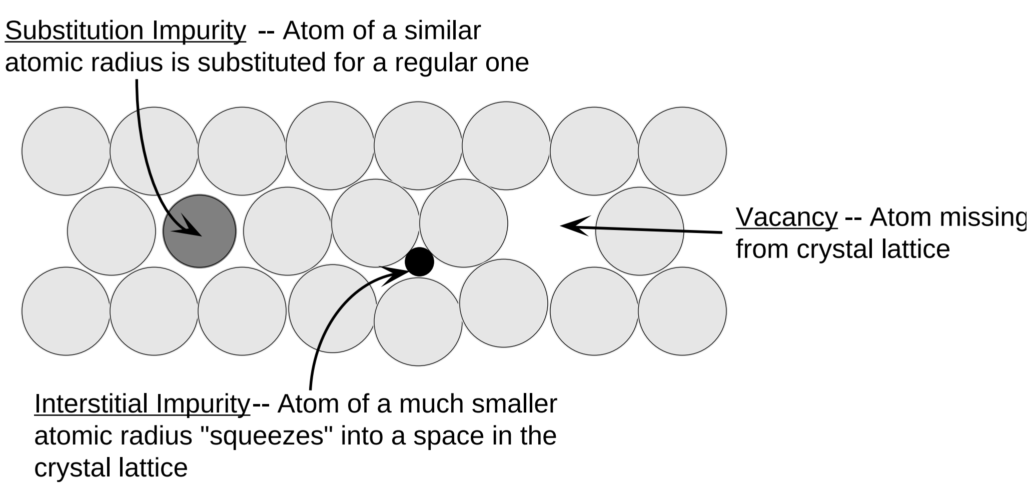
- Point
- Linear
- Planar

15.2.1 Point Defects

To general types of point defects are possible:

- Impurities
  - Substitution
  - Interstitial
- Vacancies

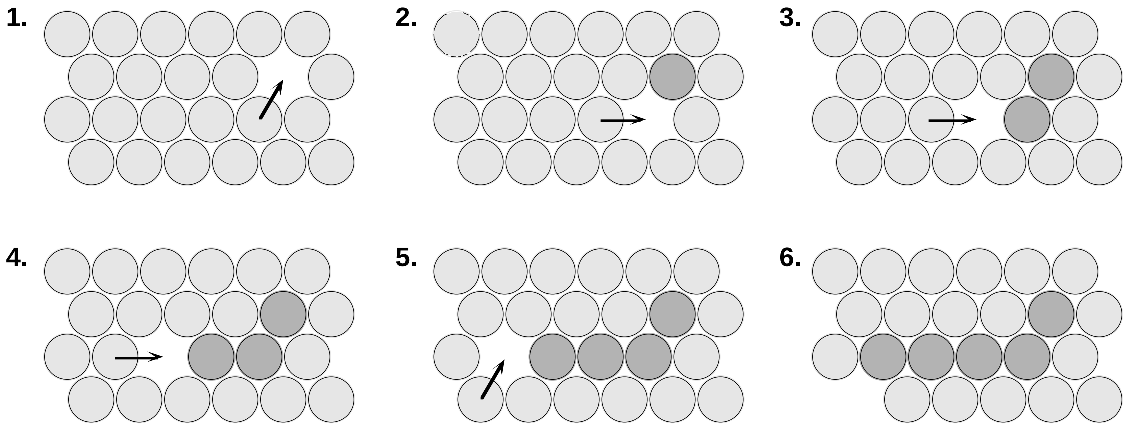
Impurities occur when a "foreign" atom is found in the crystal structure, either in place of an atom that is supposed to be there (substitution) or in the spaces between the existing atoms. Vacancies occur when an atom is missing from its normal spot in the crystal lattice, leaving a "hole". These are illustrated below:



Because the crystal does not have its ideal configuration, it has a higher internal energy and is therefore weaker than the equivalent ideal crystal.

15.2.2 Diffusion

In general, crystals contain more vacancies at higher temperature. The vacancies facilitate the movement of atoms through the crystal structure because atoms adjacent to a vacancy can "jump" into it. This general process is known as **diffusion**. This is illustrated in the following figure:



[the darker gray atoms have all moved from their original position by jumping into the adjacent vacancy. Atoms and vacancies diffuse in opposite directions]

There are two types of diffusion:

- **<u>Crystal lattice diffusion</u>** (Herring Nabarro creep) -- This type is important only at high temperatures ($T \approx 0.85\ T_{melting}$) such as one finds in the mantle of the earth because it occurs far too slowly at crustal temperatures. [shown above]

- **<u>Grain boundary diffusion</u>** (Coble creep) -- This type occurs at lower temperatures such as those found in the Earth's crust.

15.2.3 Planar Defects

There are several types of planar defects. Most are a product of the movement of dislocations. Several are of relatively limited importance and some are still poorly understood. These include:

- Deformation bands -- planar zones of deformation within a crystal

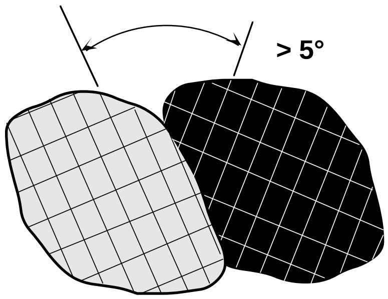
- Deformation lamellae -- similar to deformation bands; poorly understood

- Subgrain boundaries

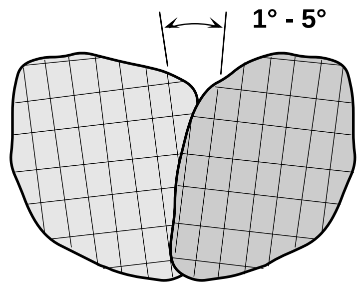
- Grain boundaries

- Twin lamellae

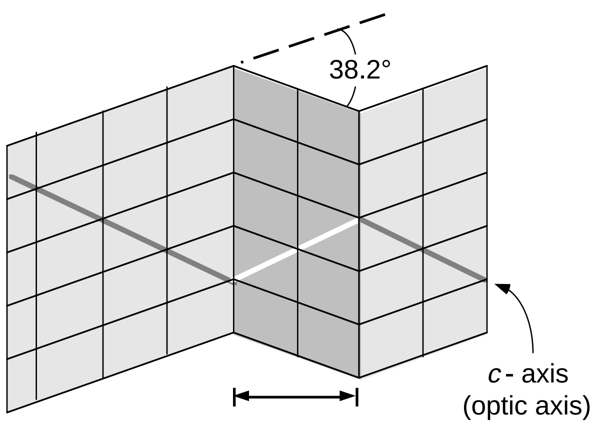
The last three are illustrated, below:



<u>Grain boundaries</u> -- ("high-angle tilt boundaries") there is a large angle mismatch of the crystal latices. This would be seen under the microscope as a large difference in extinction angles of the crystals



<u>Subgrain boundaries</u> -- ("low-angle tilt boundaries") there is a small angle mismatch of the crystal latices This would be seen under the microscope as a small difference in extinction angles of the crystals



e - lamellae in calcite
[Ca-ions at the corners of the rhombs]

<u>Twin Lamellae</u>

Narrow band in which there has been a symmetric rotation of the crystal lattice, producing a "mirror image". The twin band will have a different extinction angle than the main part of the crystal

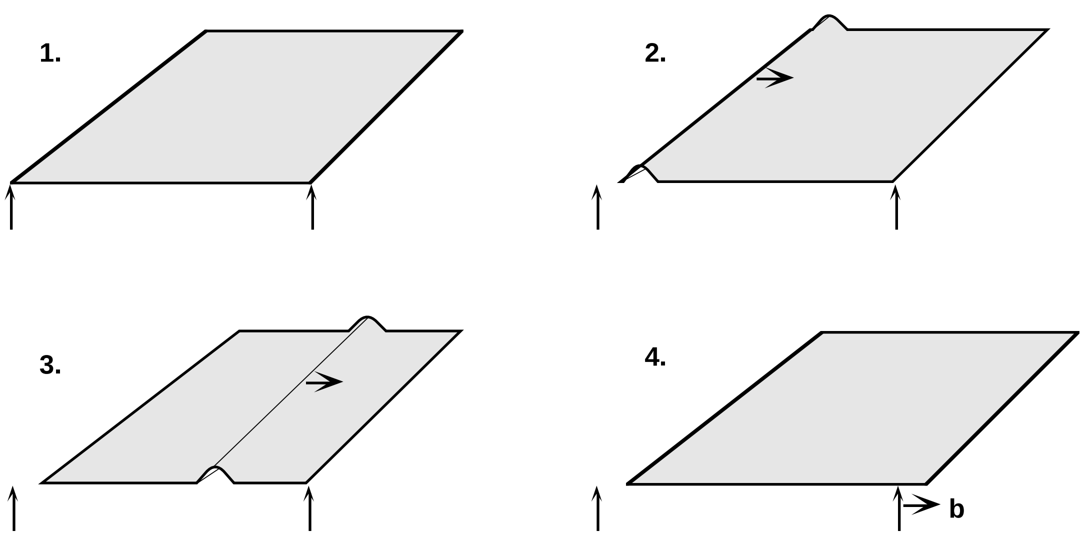
The formation of twin lamellae is called “<u>Twin gliding</u>”. This is particularly common in calcite, dolomite, and plagioclase (in which twin glide produces “albite twins”). In plagioclase, twin lamellae commonly form during crystal growth; in the carbonates, it is usually a product of deformation. Because of its consistent relationship to the crystal structure, twins in calcite and dolomite can be used as a strain gauge.

LECTURE 16—DEFORMATION MECHANISMS IV: DISLOCATIONS

16.1 Basic Concepts and Terms

Linear defects in crystals are known as **dislocations**. These are the most important defects for understanding deformation of rocks under crustal conditions. The basic concept is that it is much easier to move just part of something, a little at a time, than to move something all at once. I'm sure that that is a little obscure, but perhaps a couple of non-geological examples will help.

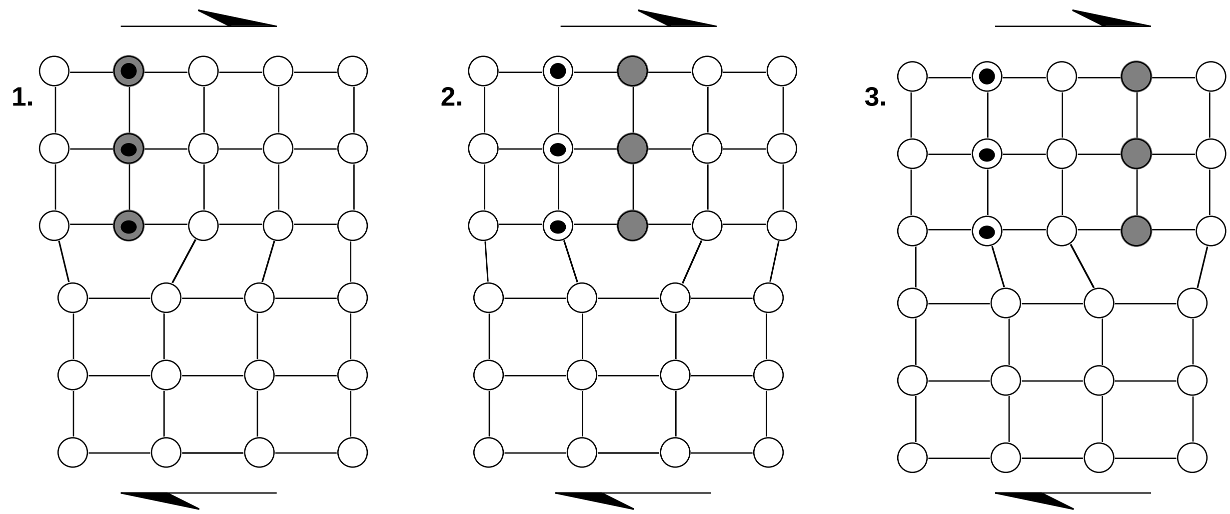
The first example is well known: How do you move a carpet across the floor with the least amount of work? If you just grab onto one side of it and try and pull the whole thing at once, it is very difficult, especially if the carpet has furniture on it, because you are trying to simultaneously overcome the resistance to sliding over the entire rug at the same time. It is much easier to make a "rumple" or a wave at one side of the rug and then "roll" that wave to the other side of the rug:

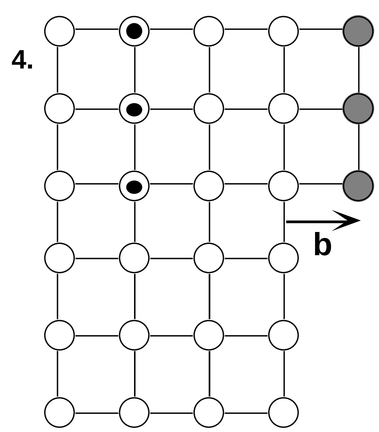


rug has now moved one full "unit" to the right

Freight trains also provide a lesser known example. A long train actually starts by backing up. There is a small amount of play in the connections between each car. After backing up, when the train moves forward for a small instant it is just moving itself, then just itself and the car behind it, etc. This way, it does not have to start <u>all</u> of the cars moving at one time.

Crystals deform in the same way. It is much easier for the crystal to just break one bond at a time than to try and break all of them simultaneously.





The the line of atoms in gray in each step represents the <u>extra half plane</u> for that step. the atoms that comprised the intial extra half plane are indicated by black dots.

The dislocation line is the bottom edge of the extra half plane. In this diagram, it is perpendicular to the page. In each step, only a singe bond is broken, so that the dislocation moves in increments of one lattice spacing each time. This distance that the dislocation moves is known as the <u>Burgers Vector</u>, and is indicated by **b** in the diagram on the left.

Note that there is <u>no record</u> in the crystal of the passage of a dislocation; the dislocation leaves a <u>perfect crystal</u> in its "wake". Thus, a dislocation is not a fault in the crystal.

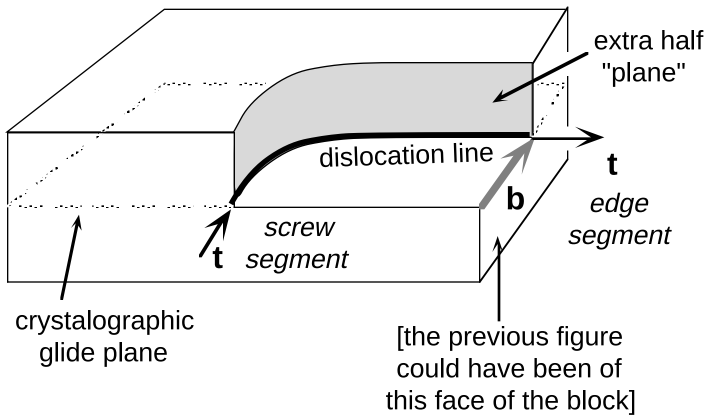
As you can see in the above figure, we describe the orientation of the dislocation and its direction of movement with two quantities:

- **Tangent vector** -- the vector parallel to the local orientation of the dislocation line

- **Burgers vector** -- slip vector parallel to the direction of movement. It is directly related to the crystal lattice spacing

These two quantities allow us to define two end member types of dislocations:

"Cut-away" view of part of a dislocation loop



Edge dislocation: **b** $\perp$ **t**

Screw dislocation: **b** // **t**

Most dislocations are closed loops which have both edge and screw components locally.

16.2 Dislocation ("Translation") Glide

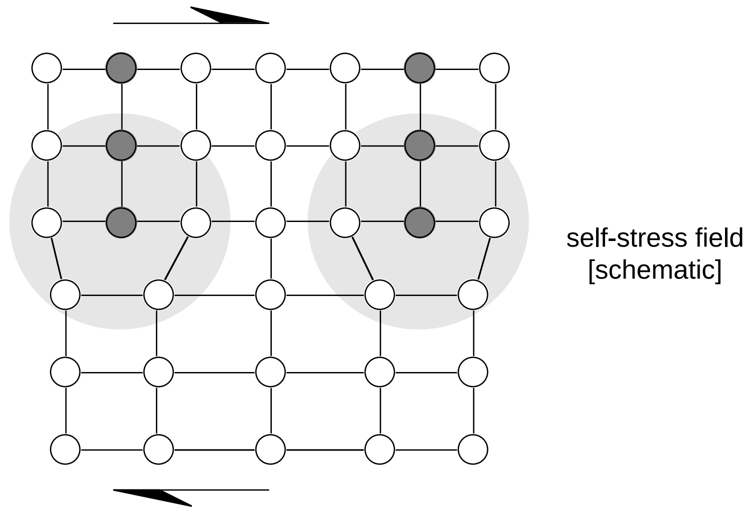
When the movement of a dislocation is confined to a single, crystallographically determined plane, it is known as dislocation glide (or translation glide by some). A particular crystallographic plane combined with a preferred slip direction is called a slip system.

The number of slip systems in a crystal depends on the symmetry class of the crystal. Crystals with high symmetry will have many slip systems; those with low symmetry will have fewer. Slip will start on planes with the lowest critical resolved shear stress. That is, slip will start on planes where the bonds are easiest to break.

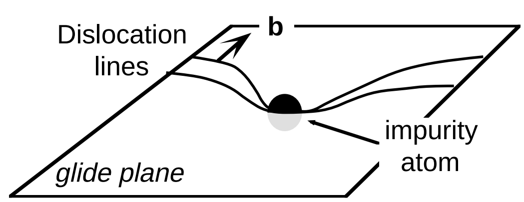
16.3 Dislocations and Strain Hardening

After dislocations begin to move or glide in their appropriate slip planes, there are three things that happen almost immediately which make it more difficult for them to continue moving:

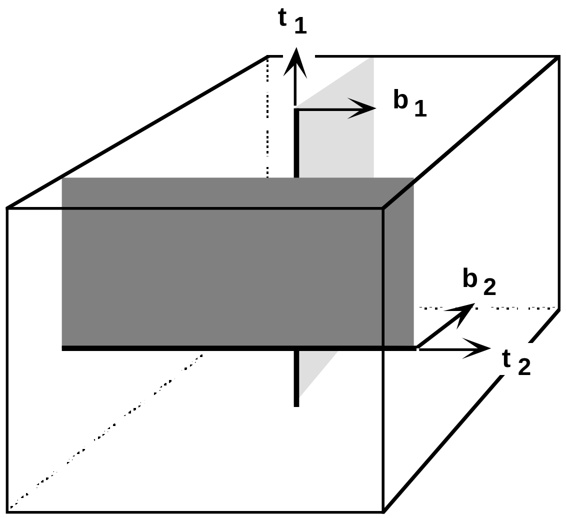
1. **Self stress field**: there is a stress field around each dislocation line which is related to the elastic distortion of the crystal around the extra half plane. In this case, the dislocations repell each other so that it takes more stress to get them to move:



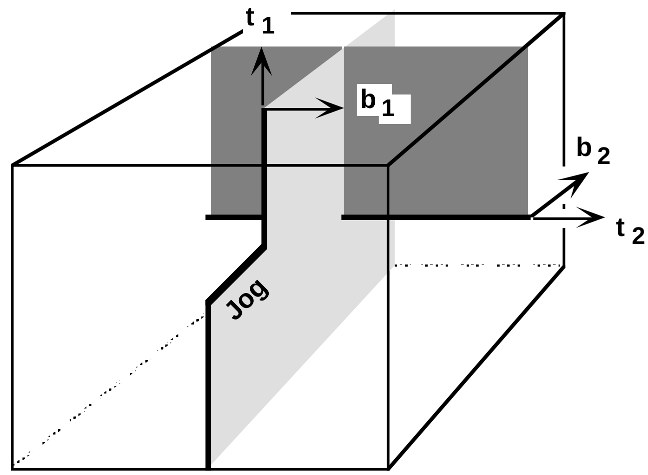
2. **Dislocation Pinning (pile-up)**: This occurs when an impurity point defect lies in the glide plane of a dislocation. If the impurity atom is tightly bound in the crystal lattice, the dislocation, which is everywhere else in its glide plane slipping freely, will become pinned by the atom. Other dislocations in the same glide plane will also encounter the same impurity, and will tend to pile up at that point.



3. **Jogs**: When dislocations of different slip systems pass through each other, one produces a jog or step in the other. This jog makes it much more difficult to move because the "jogged" segment quite probably requires a different critical resolved shear stress to move. In the diagram, below, the extra half planes are shown in shade of gray:



Before the two dislocations
run into each other



After they pass, a jog has been
produced in dislocation 1

16.4 Dislocation Glide and Climb

If there are a sufficient number of vacancies in a crystal, when a dislocation encounters an impurity atom in its glide plane the dislocation can avoid being pinned by jumping to a parallel crystal-lographic plane. This jump is referred to as dislocation climb.

The process of dislocation climb is markedly facilitated by the *diffusion* of vacancies through the crystal. Thus, climb occurs at higher temperatures because there are more vacancies at higher temperatures.

It is important to understand that diffusion has two roles in deformation: It can be the primary deformation mechanism (but probably only in the mantle for crystal lattice diffusion), or it can aid the process of dislocation glide and climb.

When dislocation glide and climb occurs, strain hardening no longer takes place. The material either acts as a perfect plastic, or it strain softens.

There are several new terms that can be introduced at this point:

Cold Working -- Plastic deformation with strain hardening. The main process is dislocation glide.

Hot working -- Permanent deformation with little or no strain hardening or with strain softening.

The main process is dislocation glide and climb.

Annealing -- Heating up a cold worked, strain hardened crystal to the point where diffusion becomes rapid enough to permit the glide and climb of dislocations. Then the dislocations either climb out of the crystal, into sub-grain walls, or they cancel each other out, producing a strain free grain from one that was obviously deformed and strained.

16.5 Review of Deformation Mechanisms

- Elastic deformation -- Very low temperature, small strains
- Fracture -- Very low temperature, high differential stress
- Pressure Solution -- Low temperature, fluids necessary
- Dislocation glide -- Low temperature, high differential stress
- Dislocation glide and climb -- Higher temperature, high differential stress
- Grain boundary diffusion -- Low temperature, low differential stress
- Crystal lattice diffusion -- High temperature, low differential stress

Lecture 17—Flow Laws & State of Stress in the Lithosphere

Experimental work over the last several years has provided data which enable us to determine how stress and strain -- or more specifically stress and strain *rate* -- are related for crystal plastic mechanisms. The relationship for dislocation glide and climb is known as **power law creep**, for diffusion, **diffusion creep**.

17.1 Power Law Creep

The basic equation which governs dislocation glide and climb is:

$$\dot{e} = C_o(\sigma_1 - \sigma_3)^n \exp\left(\frac{-Q}{RT}\right). \qquad (17.1)$$

The variables are:

$\dot{e}$ = strain rate [s^{-1}]
C_o = a constant [$GPa^{-n}s^{-1}$; experimentally determined]
$\sigma_1 - \sigma_3$ = the differential stress [GPa]
n = a power [experimentally determined]
Q = the activation energy [kJ / mol; experimentally determined]
R = the universal gas constant = 8.3144×10^{-3} kJ / mol °K
T = temperature, °K [°K = °C + 273.16°]

It is called "power law" because the strain rate is proportional to a power of the differential stress. Because temperature occurs in the exponential function, you can see that this sort of rheology is going to be *extremely* sensitive to temperature. To think of it another way, over a very small range of temperatures, rocks change from being very strong to very weak. The exact temperature at which this occurs depends on the lithology.

Using this equation and the data from Appendix B in Suppe (1985) you can easily calculate the differential stress that aplite can support at 300°C assuming that power law creep is the deformation mechanism. First of all, rearranging the above equation:

$$\sigma_1 - \sigma_3 = \left(\frac{\dot{e}}{C_o \exp\left(\frac{-Q}{RT}\right)} \right)^{\frac{1}{n}}$$

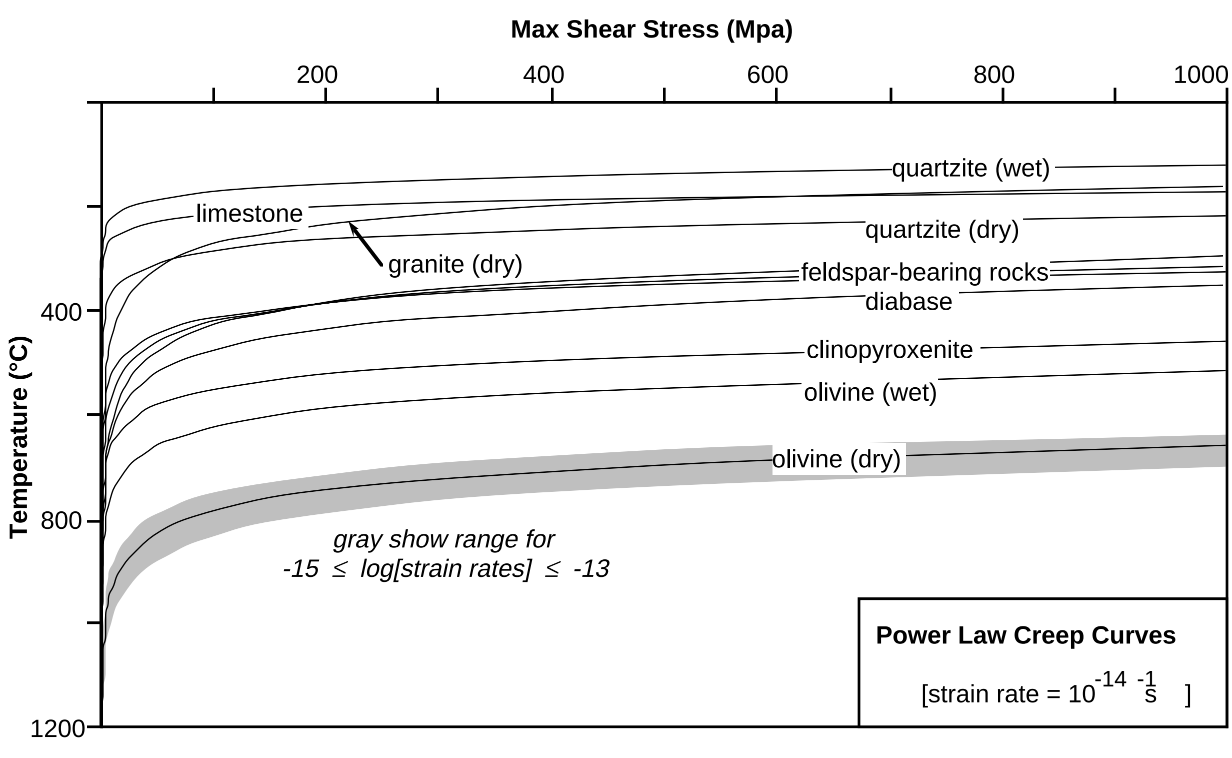
Substituting in the actual values:

$$\sigma_1 - \sigma_3 = \left(\frac{10^{-14}\mathrm{s}^{-1}}{\left(10^{2.8}\mathrm{GPa}^{-3.1}\mathrm{s}^{-1}\right)\exp\left(\frac{-163\ \mathrm{kJ\ mol}^{-1}}{8.3144\times10^{-3}\ \mathrm{kJ\ mol}^{-1}\ \mathrm{K}^{-1}(273.16+300)\ \mathrm{K}}\right)} \right)^{\frac{1}{3.1}}$$

After working through the math, you get:

$$\sigma_1 - \sigma_3 = 0.236\ \mathrm{GPa} = 236\ \mathrm{MPa}\ .$$

These curves can be constructed for a variety of rock types and temperature (just by iteratively carrying out the same calculations we did above), and we get the following graph of curves:



Note that, for a geothermal gradient of 20°C / km and a 35 km thick continental crust, the temperature at the base of the crust would be 700°C; there, only olivine would have significant strength.

17.2 Diffusion Creep

This mechanism is a linear function of the differential stress and is more sensitive to grain size than temperature:

$$\dot{e} = C_o(T)\frac{D(\sigma_1 - \sigma_3)}{d^n} \ . \tag{17.2}$$

Again, the variables are:

$\dot{e}$ = strain rate [s^{-1}]
$C_o(T)$ = a temperature dependent constant [experimentally determined]
$\sigma_1 - \sigma_3$ = the differential stress
n = a power [experimentally determined]
D = the diffusion coefficient [experimentally determined]
d = the grain size

In diffusion, the strain rate is inversely proportional to the grain size. Thus, the higher the grain size, the slower the strain rate due to diffusional processes. Although crystal lattice diffusion requires high overall temperatures, it is not nearly so sensitive to changes in temperature.

17.3 Deformation Maps

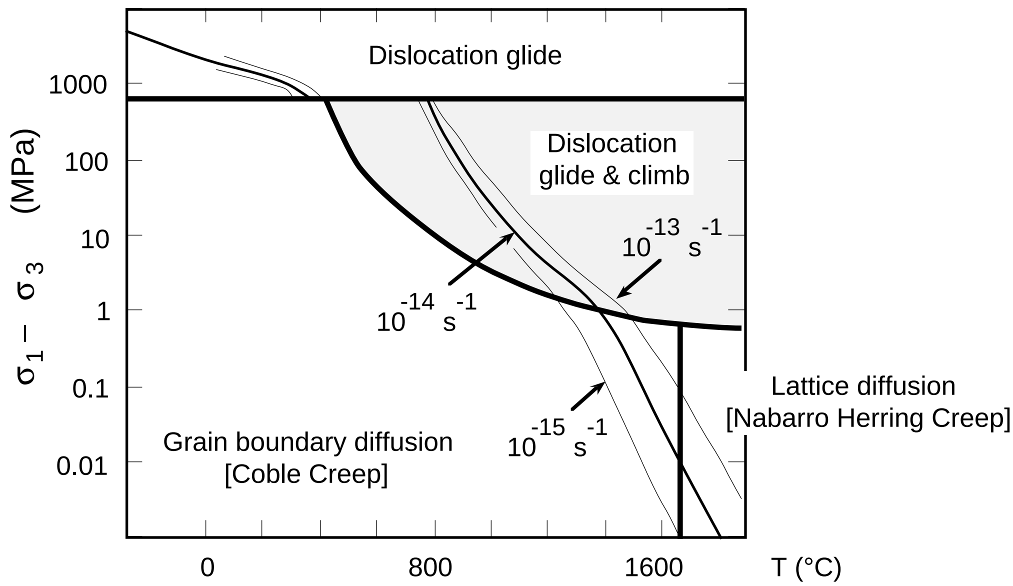
With these flow laws, we can construct a diagram known as a deformation map, which shows what deformation mechanism will be dominant for any combination of strain rate, differential stress, temperature, and grain size. Generally there are two types:

- differential stress is plotted against temperature for a constant grain size; different curves on the diagram represent different strain rates.

- differential stress is plotted against grain size for a constant temperature;

different curves on the diagram represent different strain rates. This one is generally easier to construct.

The diagram below shows an example of the first type for the mineral olivine.



17.4 State of Stress in the Lithosphere

By making a number of assumptions, we can use our understanding of the various deformation mechanisms and their related empirically derived stress-strain relations (or flow laws) to predict how stresses vary in the earth's crust. Four basic assumptions are made; two relate to the deformation mechanisms and two relate to the lithologies:

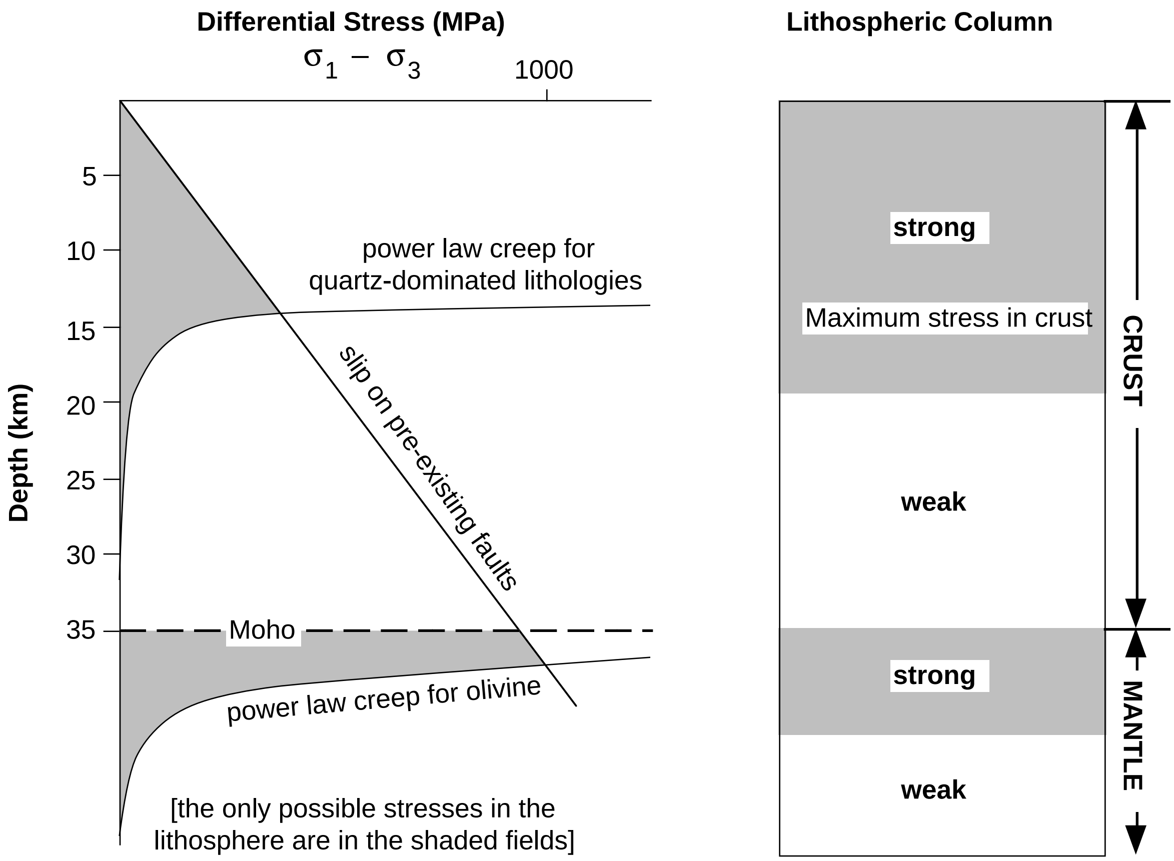
- The upper crust is dominated by slip on pre-existing faults. Thus we will use the Coulomb relation for the case of zero cohesion:

$$\sigma_s = \sigma_n * \mu_s . \tag{17.3}$$

- The lower crust is dominated by the mechanism of power law creep as described by the equation developed above (eqn 17.1).

- The crust is dominantly composed of quartz and feldspar bearing rocks.
- The mantle is composed of olivine.

The basic idea is that the crust will fail by whatever mechanism requires less differential shear stress. [Remember that the maximum shear stress is just equal to one-half the differential stress.] The resulting curve has the following form:



This model is sometimes humorously referred to as the "jelly sandwich" model of the crust. It predicts that the lower crust will be very weak (supporting differential stresses of < 20 Mpa) relative to the upper crust and upper mantle; it will behave like jelly between two slices of (stiff) bread. In general, the most numerous and the largest earthquakes tend to occur in the region of the stress maximum in the middle crust, providing at least circumstantial support to the model.

These curves are sometimes incorrectly referred to as "brittle-ductile transition" curves. Because we have used very specific rheologies to construct them, they should be called "<u>frictional crystal-plastic</u>

transition" curves.

Now, we should review some of the important "hidden" assumptions and limitations of these curves, which have been very popular during the last decade:

- *Lithostatic load and confining pressure control the deformation of the upper crust* -- notice that there is no depth term in eqn. 17.3, even though the vertical axis of the graph is plotted in depth. The depth is calculated by assuming that the vertical stress is either σ_1 or σ_3 and that it is equal to the lithostatic load:

$$\sigma_1 \text{ or } \sigma_3 = P_{lith} = \rho g z$$

- *Temperature is the fundamental control on deformation in the lower crust* -- Again, there is no depth term in the Power Law Creep equation (17.1). Depth is calculated by assuming a geothermal gradient and calculating the temperature at that depth based on the gradient. *So really, two completely different things are being plotted on the vertical axis and neither one is depth!*

- *Friction is assumed to be the main constraint on deformation in the upper crust* -- The value of friction is assumed to be constant for all rock types. [This follows from "Byerlee's Law" which we will discuss in a few days.]

- *Laboratory strain rates are extrapolated over eight to ten orders of magnitude to get the power law creep curves* -- the validity of this extrapolation is not known.

- *Other deformation mechanisms are not considered to be important* -- The most important of these would include pressure solution, the unknown role of fluids in the lower crust, and diffusion.

- *There is a wide variation in laboratory determined constants for all of the flow laws* -- Basically, do not take the specific numbers too seriously.

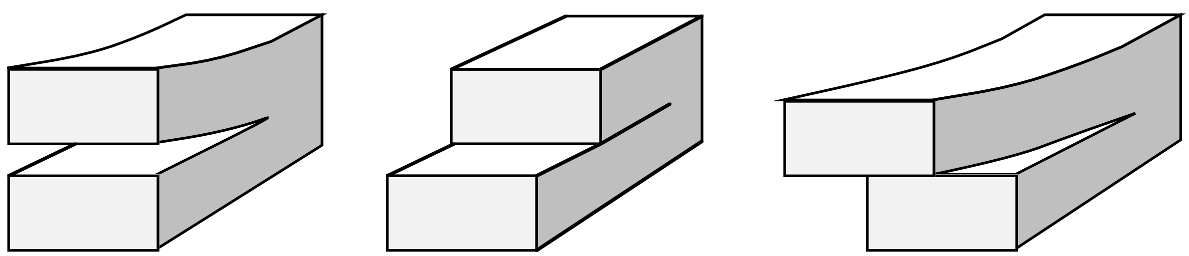
LECTURE 18—JOINTS & VEINS

18.1 Faults and Joints as Cracks

We'll start our exploration of structures with discontinuous structures and later move on to continuous structures. There are two basic types of discontinuous structures:

- **Faults** -- discontinuities in which one block has slipped past another, and
- **Joints** -- where block move apart, but do not slip past each other.

Most modern views of these structures are based on crack theory, which we had some exposure to previously when we talked about the failure envelop. There are three basic "modes" of cracks:



Mode I : opening **Mode II** : sliding **Mode III** : tearing

Looked at this way, faults are mode II or mode III cracks, while joints are mode I cracks. Notice the gross similarity between mode II cracks and edge dislocations and mode III cracks and screw dislocations. Although they are similar, bear in mind that there are major differences between the two.

Definition of a joint: a break in the rock across which there has been no shearing, only extension. Basically, they are mode I cracks. If it is not filled with anything, then it is called a joint; if material has been precipitated in the break, then it is called a vein.

18.2 Joints

Joints are characteristic features of all rocks relatively near the Earth's surface. They are of great practical importance because they are pre-fractured surfaces. They have immediate significance for:

- mining and quarrying

- civil engineering
- ground water circulation
- hydrothermal solutions and mineral deposits

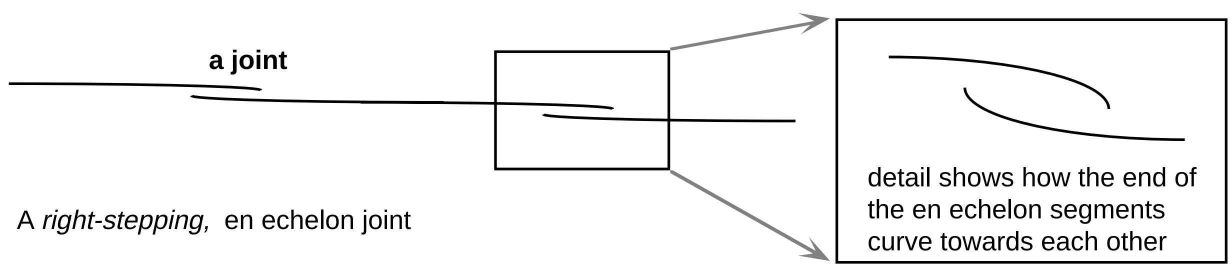
Despite their ubiquitous nature and their practical importance, there are several reasons why analyzing joints is not easy and is subject to considerable uncertainty:

- age usually unknown
- they are easily reactivated
- they represent virtually no measurable strain
- there are many possible mechanisms of origin

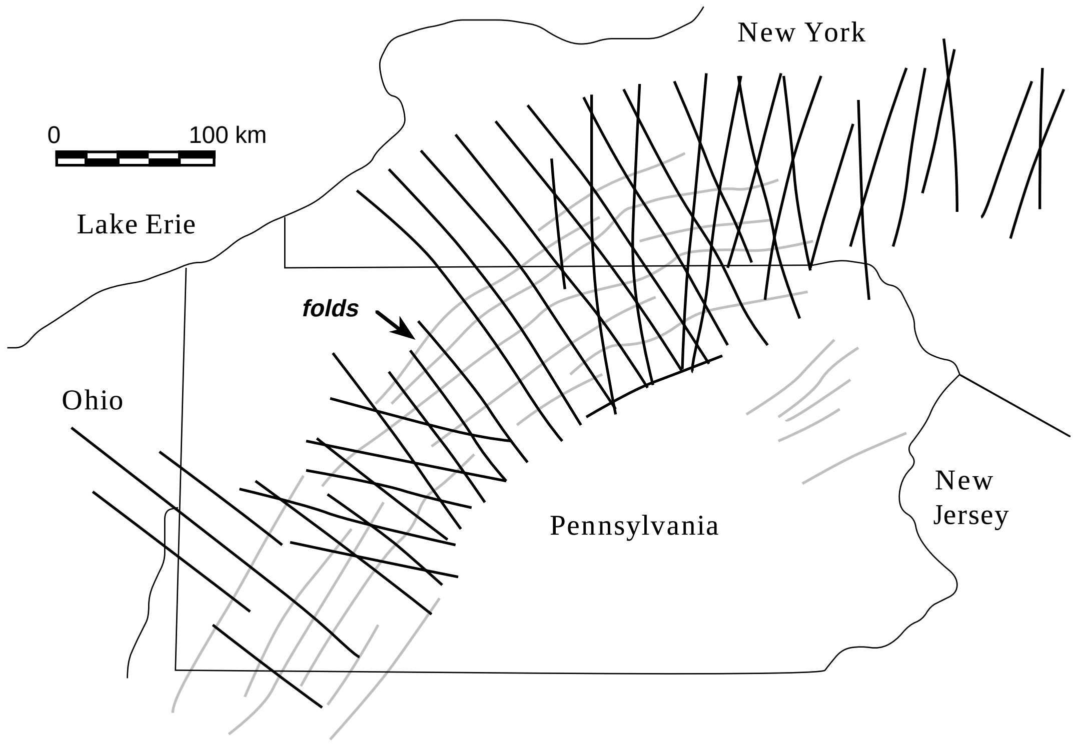
18.2.1 Terminology

Systematic joints commonly are remarkably smooth and planar with regular spacing. They nearly always occur in sets of parallel joints. **Joint sets** are systematic over large regions. **Joint systems** are composed of two or more joint sets. Joints which regularly occur between (i.e. they do not cross) two member of a joint set are called **cross joints**.

Most joints are actually a **joint zone** made of *"en echelon"* sets of fractures:

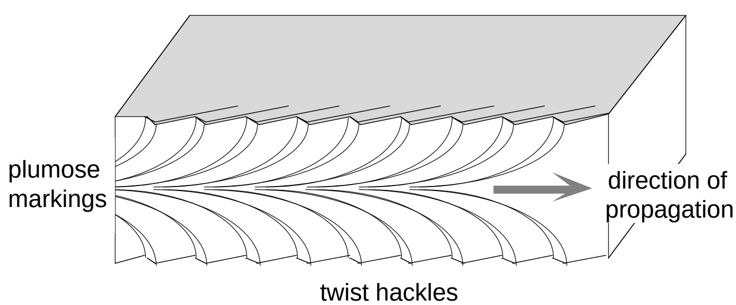


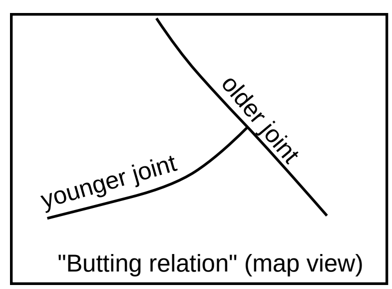
Joint systems are consistent over large regions indicating that the scale of processes that control jointing is also regional in nature. For example, in the Appalachians, the joints are roughly perpendicular to the fold axes over broad regions:



Joints are not always perpendicular to fold axes or even related to regional folds in any systematic way. On the Colorado Plateau, for example, joints in sedimentary rocks are parallel to the metamorphic foliation in the basement.

18.2.2 Surface morphology of the joint face:



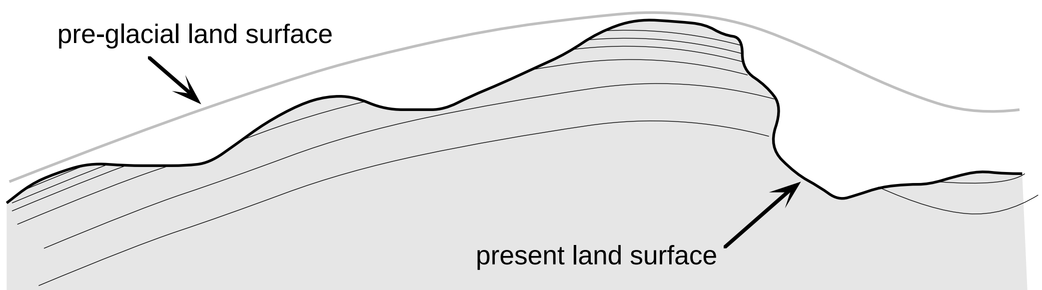


This kind of morphology indicates that the fracture propagates very rapidly. Younger joints nearly always terminate against older joints at right angles. This is called a **butting relation**. As we will see later in the course, this occurs because the older joint acts like as free surface with no shear stress

18.2.3 Special Types of Joints and Joint-related Features

Although many joints are tectonic in origin (e.g. the joints in sedimentary rocks of the Ithaca area), others are totally unrelated to tectonics. Some special types:

Sheet structure or exfoliation -- This is very common in the granitoid rocks and other rocks are were originally free from other types of joints. Sheet joints form thin, curved, generally convex-upward shells which parallel the local topography. The sheets get thicker and less numerous with depth in the earth and die out completely at about 40 m depth. The sheets are generally under compression parallel to their length; the source of this compression is not well understood. In general, they are related to gravitational unloading of the granitoid terrain. In New England, they have been used to construct the pre-glacial topography because they formed before the last glaciation:



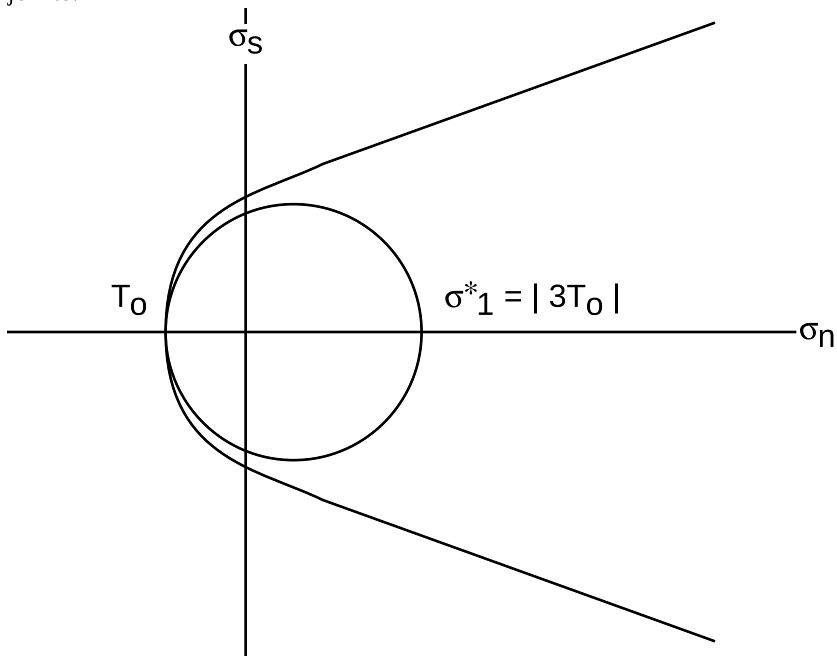
Spalling and rock bursts in mines and quarries -- In man made excavations, the weight of the overburden is released very suddenly. This creates a dangerous situation in which pieces of rock may literally “explode” off of the newly exposed wall or tunnel (it is released by the formation of a joint at acoustic velocities). For this reason quarries, especially deep ones, after miners make a new excavation, no one is allowed to work near the new face of rock for a period of hours or days until the danger of rock bursts has passed.

Cooling joints in volcanic rocks—The process involved is thermal contraction; as the rock cools it shrinks, pulling itself apart. This is the source of the well known columnar joints in basaltic rocks, etc.

18.2.4 Maximum Depth of True Tensile Joints

True tensile joints, with no shear on their surfaces, occur only in the very shallow part of the

Earth's crust. The shape of the Mohr failure envelop gives us some insight into the maximum depth of formation of true joints:



If we assume that, near the surface of the earth, σ_1 is vertical, then we can write the stress as a function of depth, the density of rocks, and the pore fluid pressure:

$$\sigma_1 = \rho g z\ (1 - \lambda)$$

where λ is the fluid pressure ratio: $\lambda = P_f\ /\ \rho g z$.

The maximum depth of formation of tensile joints, then, is:

$$Z_{\max} = \left| \frac{3T_o}{\rho g(1-\lambda)} \right|$$

Thus, except at very high pore fluid pressures, the maximum depth of formation of joints is about 6 km, given that the tensile strength of rocks, T_o, is usually less than 40 MPa.

18.3 Veins

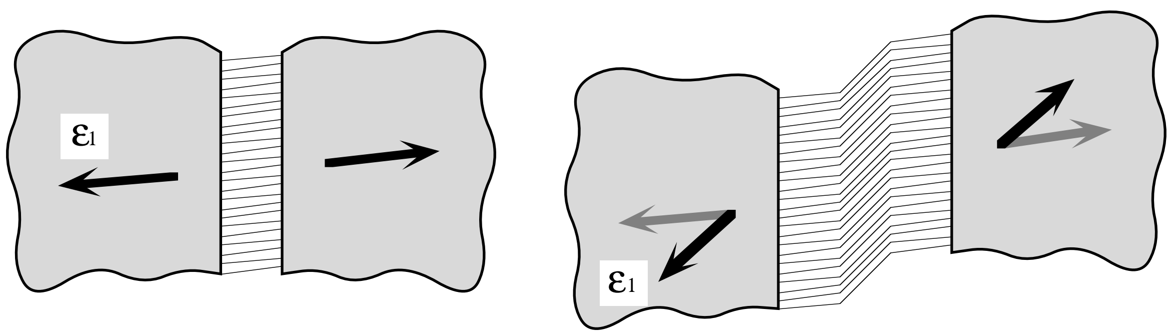
Veins form when joints or other fractures in a rock with a small amount of shear are filled with material precipitated from a fluid. For many reasons, veins are extremely useful for studying local and regional deformations:

- record incremental strains
- many contain dateable material
- fluid inclusions in the vein record the temperature and pressure conditions at the time the vein formed

In addition, veins have substantial economic importance because many ore deposits are found in veins. The Mother Lode which caused the California gold rush in 1849 is just a large gold-bearing quartz vein.

18.3.1 Fibrous Veins in Structural Analysis

An extremely useful aspect of many veins is that the minerals grow in a fibrous form as the walls of the vein open up, with the long axes of the fibers parallel to the incremental extension direction.

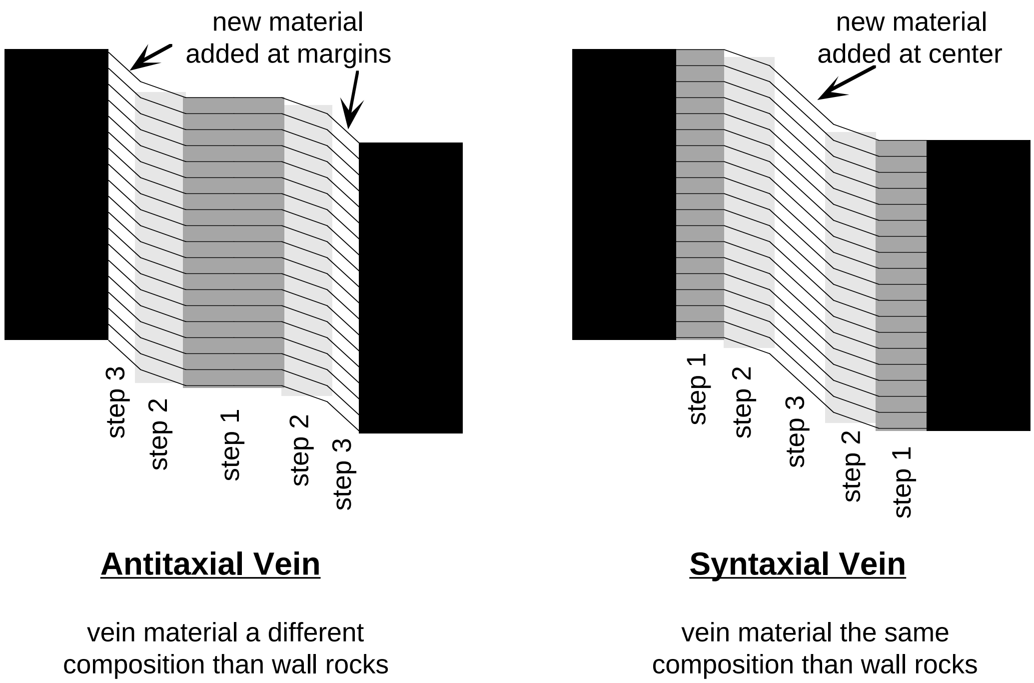


Step 1 **Step 2**

There are two types of fibrous veins, and it is important to distinguish between them in order to use them in structural analysis:

Syntaxial veins form when the vein has the same composition as the host rock (e.g. calcite veins in limestone). The first material nucleates on crystals in the wall of the vein and grows in optical continuity with those. New material is added at the center of the vein (as in the example, above).

Antitaxial veins form when the vein material is a different composition than the host rock (e.g. calcite vein in a quartzite). New material is always added at the margins of the vein.



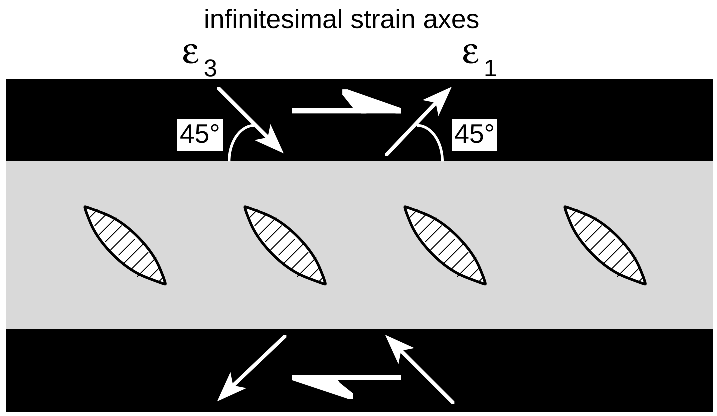
These are among the very few natural features which show the rotational history of a deformation and thus are particularly useful for studying simple shear deformations.

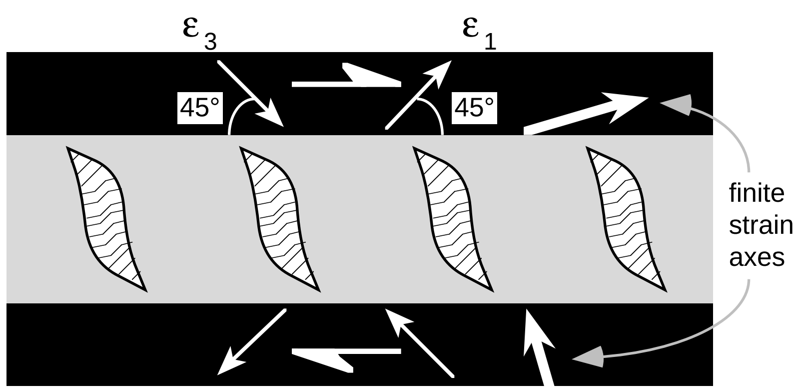
It is important to remember that the fibers are <u>not deformed</u>. They are simply growing during the deformation.

18.3.2 En Echelon Sigmoidal Veins

Veins in which the tip grows during deformation (so that the entire vein gets larger) also provide information on the incremental history of the deformation. The tip always grows perpendicular to the incremental (or infinitesimal) principal extension), even though the main part of the vein may have rotated during the simple shear. These veins are called <u>sigmoidal veins</u> or sometimes “tension gashes.” They can also be syntaxial or antitaxial, thus providing even more information.

The formation of all of these types of veins in a simple shear zone is illustrated below:





Recall that, in a shear zone, the axes of the infinitesimal strain ellipse are always oriented at 45° to the shear plane. Because the tips of the sigmoidal veins always propagate perpendicular to the *infinitesimal* extension direction, the tips will also be at 45° to the shear zone boundary. If the veins grow in a syntaxial style, as in the above diagram, the fibers at the tips and in the center of the vein will also be at 45°.

18.4 Relationship of Joints and Veins to other Structures

Faults& Shear Zones

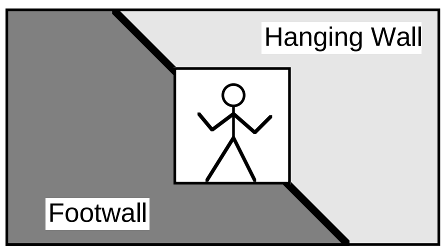
Folds

LECTURE 19—FAULTS I: BASIC TERMINOLOGY

19.1 Descriptive Fault Geometry

For faults that are not vertical, there are two very useful terms for describing the blocks on either side of the fault. These terms can be used either for normal or reverse faults:

- **Hanging Wall**, so called because it "hangs" over the head of a miner, and
- **Footwall**, because that's the block on which the miners feet were located.



The three dimensional geometry of a fault surface can be quite variable, and there are several terms to describe it:

- **Planar** -- a flat, planar surface
- **Listric** (from the Greek word "*listron*" meaning shovel shaped) -- fault dip becomes shallower with depth, i.e. concave-upward
- **Steepening downward** or convex up
- **Anastomosing** -- numerous branching irregular traces

In three dimensions, faults are irregular surfaces. All faults either have a point at which (a) their displacement goes to zero, (b) they reach a point where the intersect another fault, or (c) they intersect the surface of the Earth. There are three terms to describe these three possibilities:

- **Tip Line** -- Where fault displacement goes to zero; it is the line which separates slipped from unslipped rock, or in the above crack diagrams, it is the edge of the crack. Unless it intersects the surface of the Earth or a branch line, the tip line is a closed loop

- **Branch Line** -- the line along which one fault intersects with or branches off of another fault

- **Surface trace** -- the line of intersection between the fault surface and the land surface

19.2 Apparent and Real Displacement

The displacement of one block relative to another is known as the **slip vector**. This vector connects two points which were originally adjacent on either side of the fault. It is extremely unusual to find a geological object which approximates a point that was “sliced in half” by a fault.

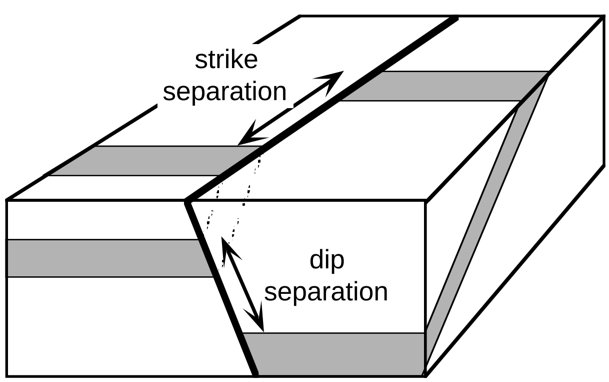
Fortunately, we can get the same information from a *linear feature* which intersects and was offset across the fault surface. Such lines are known as **piercing points**. Most such linear features in geology are formed by the intersection of two planes:

- intersection between a dike and a bed

- intersection of specific beds above and below an angular unconformity

- fold axis

It is however, much more common to see a planar feature offset by a fault. In this case, we can only talk about **separation**, not slip:



There are an infinite number of possible slips that could produce an observed separation of a planar feature. If you just saw the top of the above block, you might assume that the fault is a strike slip fault. If you just saw the front, you might assume a normal fault. However, it could be either one, or a combination of the two.

19.3 Basic Fault Types

With this basic terminology in mind, we can define some basic fault types:

19.3.1 Dip Slip

Normal -- The hanging wall moves down with respect to the footwall. This movement results in horizontal extension. In a previously undeformed stratigraphic section, this would juxtapose younger rocks against older.

High-angle -- dip > 45°

Low-angle -- dip < 45°

Reverse -- the hanging wall moves up with respect to the footwall. This movement results in horizontal shortening. In a previously undeformed stratigraphic section, this would juxtapose older rocks against younger.

High-angle -- dip > 45°

Thrust -- dip < 45°

19.3.2 Strike-Slip

Right lateral (dextral)-- the other fault block (i.e. the one that the viewer is not standing on) appears to move to the viewers right.

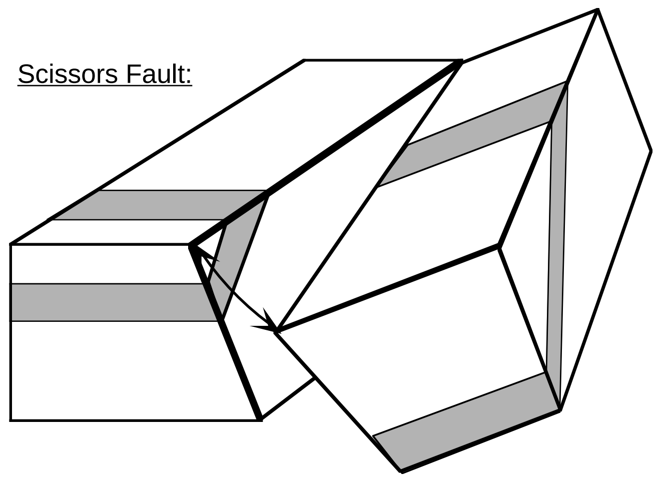
Left lateral (sinistral)-- the other fault block appears to move to the viewers left.

A **wrench fault** is a vertical strike-slip fault.

Oblique Slip -- a combination of strike and dip slip

19.3.3 Rotational fault

In this case one block rotates with respect to the other. This can be due to a curved fault surface [rotation axis is parallel to the fault surface], or where the rotation axis is perpendicular to the fault surface. The latter case produces what is commonly known as a **scissors** or a **hinge fault**:



19.4 Fault Rocks

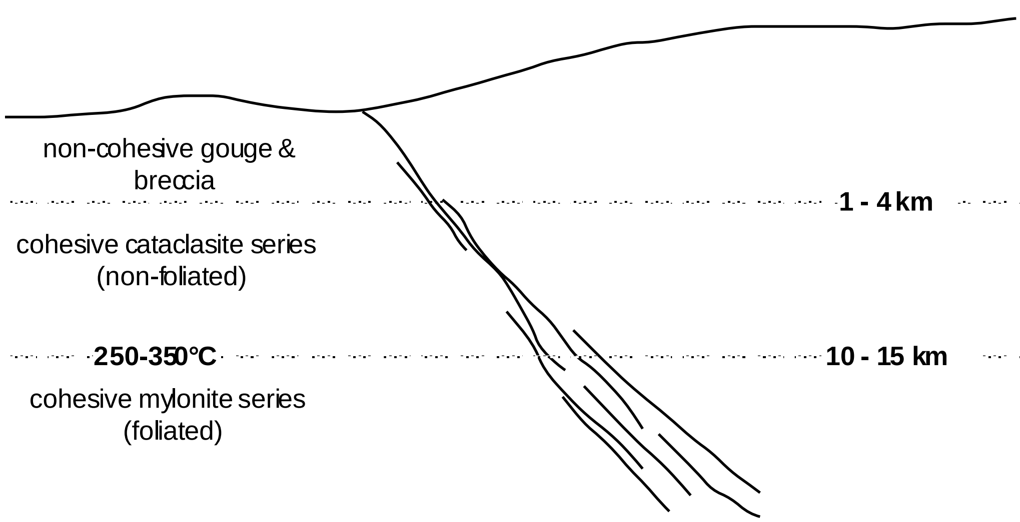
The process of faulting produces distinctive textures in rocks, and those textures can be classified according to the deformation mechanism that produced it. Again, the two general classes of mechanisms that we discussed in class are: Frictional-Cataclastic ("Brittle mechanisms"), and crystal-plastic mechanisms.

19.4.1 Sibson's Classification

Presently, the most popular classification method of fault rocks comes from the work by Sibson. He has two general categories, based on whether the texture of the rock is foliated or random:

	Random Fabric	Foliated Fabric	Proportion of Matrix
Incohesive	Fault breccia (visable fragments > 30%) Fault gouge (visable fragments < 30%)		
Cohesive	crush breccia (fragments > 0.5 cm) fine crush breccia (fragments 0.1 - 0.5 cm) crush micro-breccia (fragments < 0.1 cm)		0 - 10 %
Cohesive	Protocataclasite	Protomylonite	10 - 50 %
Cohesive	Cataclasite	Mylonite	50 - 90 %
Cohesive	Ultracataclasite	Ultramylonite	90 - 100 %

These rock types tend to form at different depths in the earth:



19.4.2 The Mylonite Controversy

There exists to this day no generally accepted definition f the term “mylonite” despite the fact that it is one of the most commonly used fault rock names. There are two or three current definitions:

- A fine grained, laminated rock produced by extreme microbrecciation and milling of rocks during movement on fault surfaces. This definition is closest to the original definition of Lapworth for the mylonites along the Moine thrust in Scotland

- Any laminated rock in which the grain size has been reduced by any mechanism during the process of faulting. This is an “intermediate” definition.

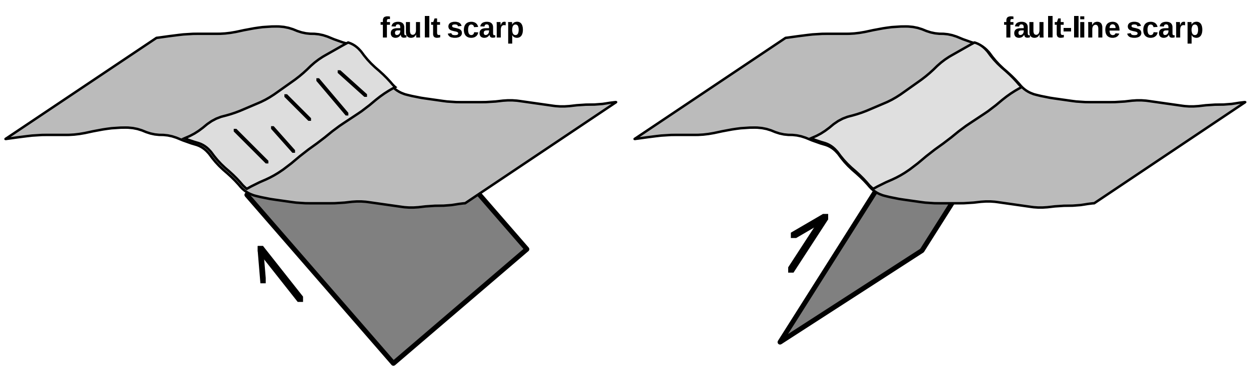
- A fault rock in which the matrix has deformed by dominantly crystal-plastic mechanisms, even though more resistant grains may deform by cracking and breaking. This definition tends to be that most used today.

The problem with these definitions is that they tend to be genetic rather than descriptive, and they don’t take into account the fact that, under the same temperature and pressure conditions, different minerals will deform by different mechanisms.

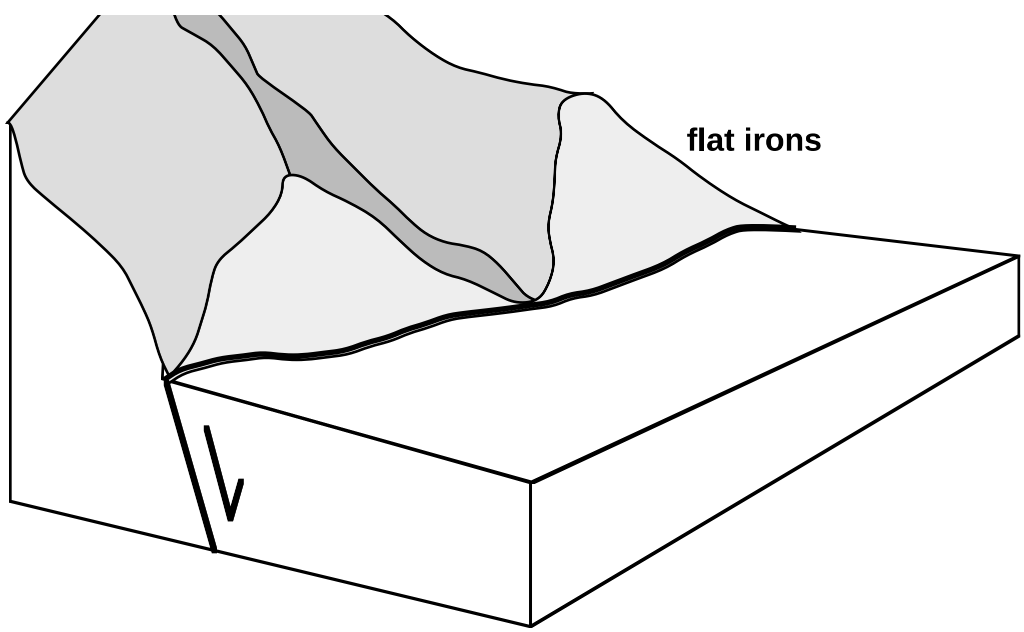
LECTURE 20—FAULTS II: SLIP SENSE & SURFACE EFFECTS

20.1 Surface Effects of Faulting

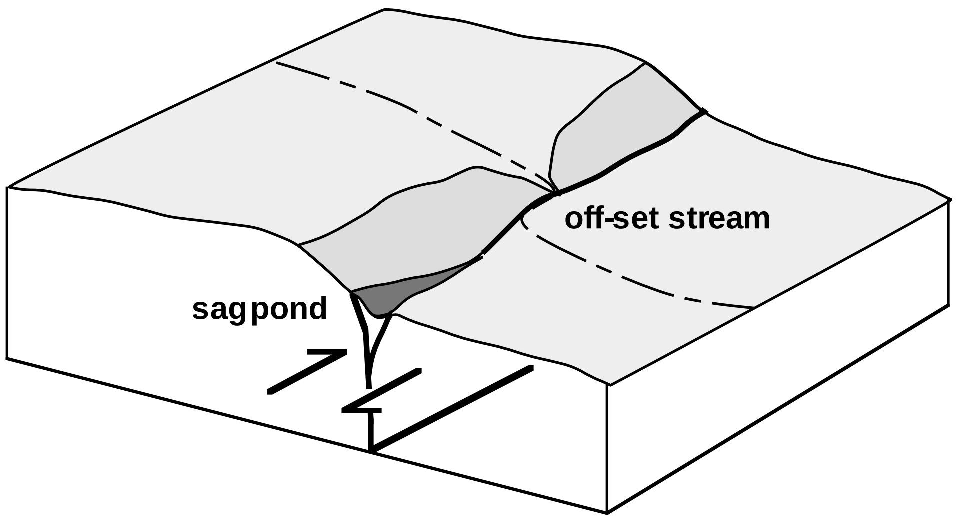
Faults that cut the surface of the Earth (i.e. the tip line intersects the surface) are known as emergent faults. They produce a topographic step known as a scarp:



The scarp can either be the surface exposure of the fault plane, in which case it is a fault scarp or it can simply be a topographic bump aligned with, but with a different dip than, the fault (a fault-line scarp). Where scarps of normal faults occur in mountainous terrain, one common geomprohic indicator of the fault line are flat irons along the moutain front:

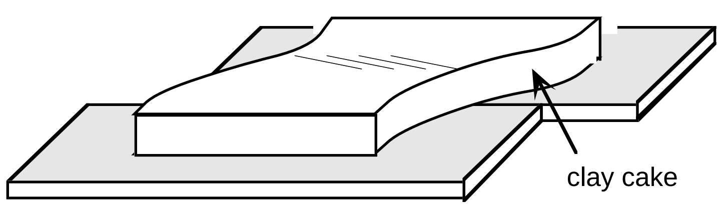


These are particularly common in the Basin and Range of the western United States. In areas of strike-slip faulting, features such as off-set stream valleys, and sag ponds — wet swampy areas along the fault trace — are common (sag ponds can also be seen along normal and thrust fault traces).



Faults which do not cut the surface of the Earth (i.e. their tip lines do not intersect the surface) are called blind faults. They can still produce topographic uplift, particularly if the tip line is close to the surface, but the uplift is broader and more poorly defined than with emergent faults. Blind faults have stirred quite a bit of interest in recent years because of their role in seismic hazard. The recent Northridge Earthquake occurred along a blind thrust fault.

20.2 How a Fault Starts: Riedel Shears

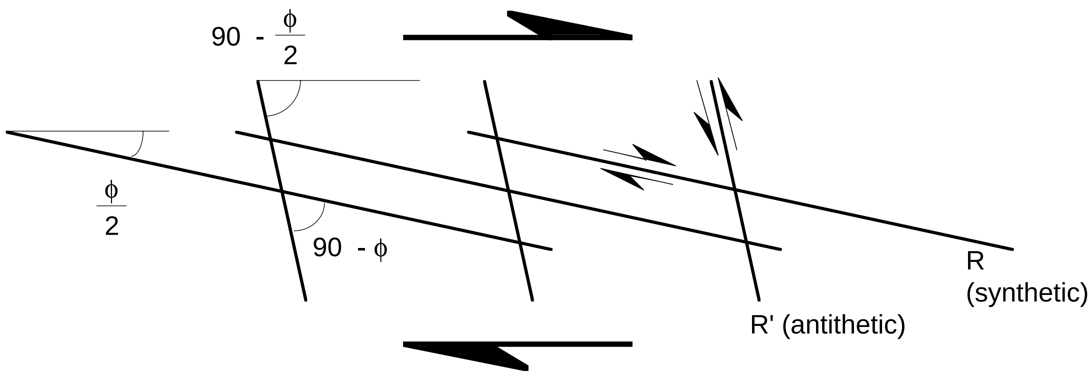


Much of our basic understanding of the array of structures that develop during faulting comes from experiments with clay cakes deformed in shear, as in the picture, above. These experiments show that strike-slip is a two stage process involving

- pre-rupture structures, and
- post-rupture structures.

20.2.1 Pre-rupture Structures

Riedel Shears:

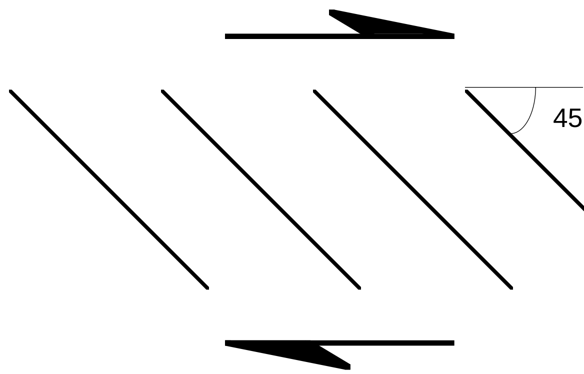


The initial angles that the synthetic and antithetic shears form at is controlled by their coefficient of internal friction. Those angles and the above geometry mean that the maximum compression and the principal shortening axis of infinitesimal strain are both oriented at 45° to the shear zone boundary.

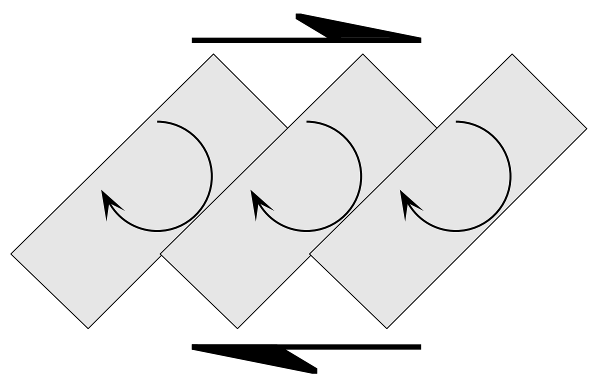
With continued shearing they will rotate (clockwise in the above diagram) to steeper angles. Because the R' shears are originally at a high angle to the shear zone they will rotate more quickly and become inactive more quickly than the R shears. In general, the R shears are more commonly observed, probably because they have more displacement on them.

Riedel shears can be very useful for determining the sense of shear in brittle fault zones.

Extension Cracks: In some cases, extension cracks will form, initially at 45° to the shear zone:



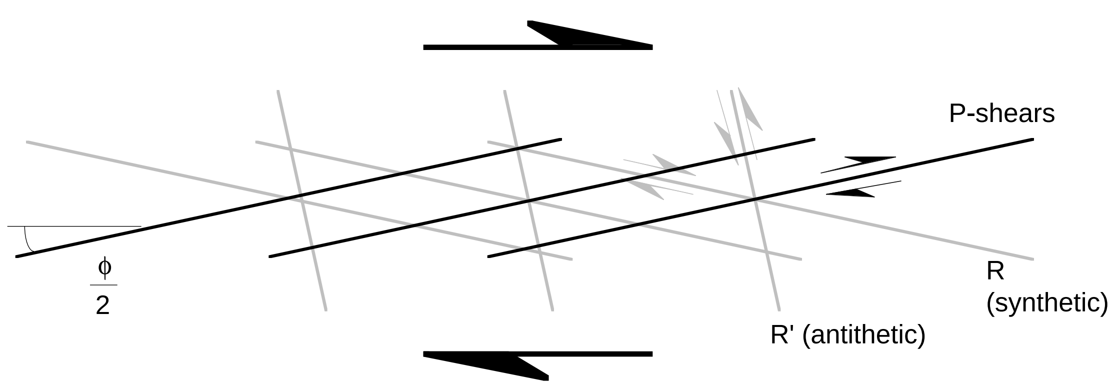
These cracks can serve to break out blocks which subsequently rotate in the shear zone, domino-style:



Note that the faults between the blocks have the opposite sense of shear than the shear zone itself.

20.2.2 Rupture & Post-Rupture Structures

A rupture, a new set of shears, called "P-shears", for symmetric to the R-shears. These tend to link up the R-shears, forming a through-going fault zone:



20.3 Determination of Sense of Slip

To understand the kinematics of fault deformation, we must determine their slip. The slip vector is composed of two things: (1) the orientation of a line along which the blocks have moved, and (2) the sense of slip (i.e. the movement of one block with respect to the other).

Geological features usually give us one or the other of these. Below, I'll give you a list of features, many of which may not mean much to you right now. Later in the course, we will describe several in detail. I give you their names now just so that you'll associate them with the determination of how a fault moves.

Orientation

Frictional-Cataclastic faults

grooves, striae, slickensides, slickenlines

Crystal plastic

mineral lineations

Sense

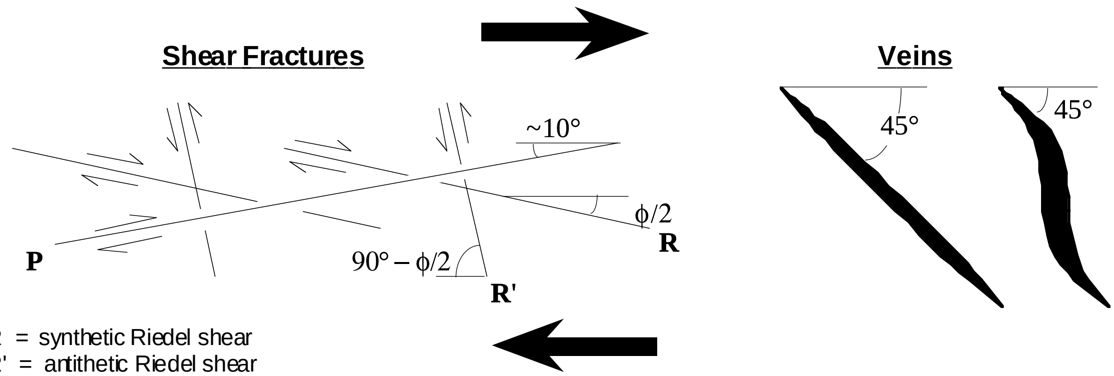
Frictional-cataclastic

Riedel shears, steps, tool marks, sigmoidal gash fractures, drag folds, curved mineral fibers

Crystal plastic mechanisms

Sheath folds, S-C fabrics, asymmetric c-axis fabrics, mica fish, asymmetric augen, fractured and rotated mineral grains

Orientations of Common Fault-Related Features

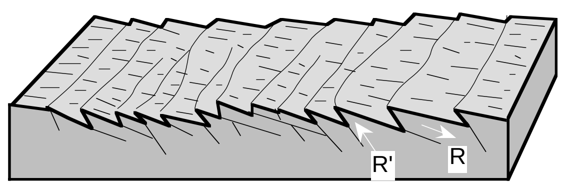


R = synthetic Riedel shear
R' = antithetic Riedel shear
P = P-shear; φ = angle of internal friction

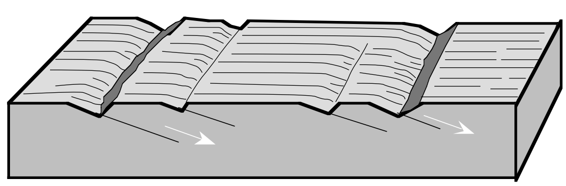
Same sense of shear applies to all following diagrams

Riedel Shears

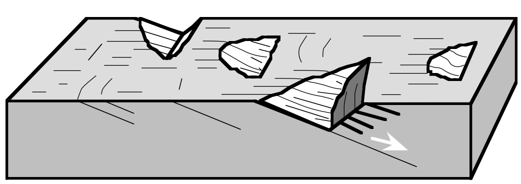
These features are well described in the classic papers by Tchalenko (1970), Wilcox et al. (1973), etc. The discussion below follows Petit (1987). It is uncommon to find unambiguous indicators of movement on the R or R' surfaces and one commonly interprets them based on striation and angle alone In my experience, R shears can be misleading and one should take particular care in using them without redundant indicators or collaborative indicators of a different type.



"RO"-Type (top): The fault surface is totally composed of R and R' surfaces. There are no P surfaces or an average surface of the fault plane. Fault surface has a serrated profile. Not very common.



"RM"-Type (middle): The main fault surface is completely striated. R shears dip gently (5-15°) into the wall rock; R' shears are much less common. The tip at the intersection of R and the main fault plane commonly breaks off, leaving an unstriated step.



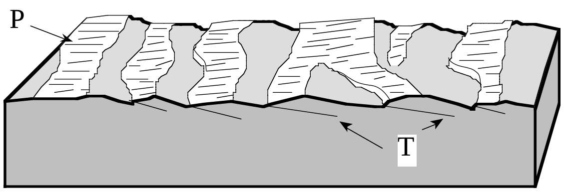
Lunate fractures (bottom): R shears commonly have concave curvature toward the fault plane, resulting in "half moon" shaped cavities or depressions in the fault surface.

diagrams modified after Petit (1987)

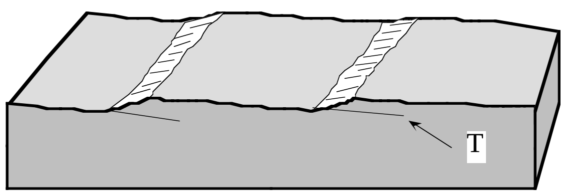
[sense of shear is top (missing) block to the right in all the diagrams on this page]

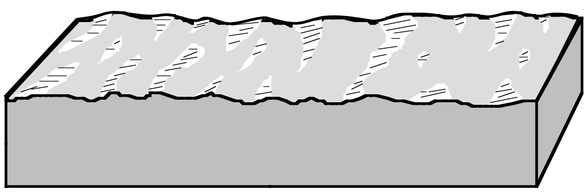
Striated P-Surfaces

These features were first described by Petit (1987). The fault plane is only partially striated, and the striations only appear on the up-flow sides of asperities.



"PT"-Type (top & middle): ~ planar, *non-striated* surfaces dip gently into the wall rock. Petit (1987) calls these "T" surfaces because of lack of evidence for shear, but they commonly form at angles more appropriate for R shears. Striated P surfaces face the direction in which that block moved. Steep steps developed locally at intersection between P and T. P surfaces may be relatively closely spaced (top) or much farther apart (middle).



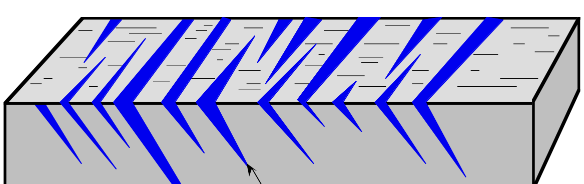


"PO"-Type (bottom) : T surfaces are missing entirely. Striated P surfaces face in direction of movement of the block in which they occur. Lee side of asperities are unstriated.

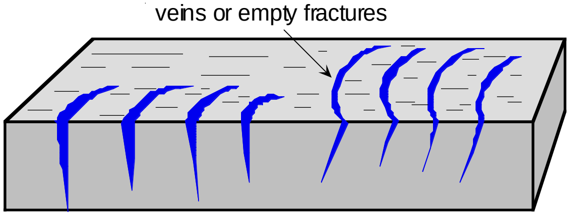
diagrams modified after Petit (1987)

Unstriated Fractures ("T fractures")

Although "T" refers to "tension" it is a mistake to consider these as tensile fractures. They commonly dip in the direction of movement of the upper (missing) block and may be filled with veins or unfilled.



"Tensile Fractures" (top): If truely tensile in origin and formed during the faulting event, these should initiate at 45° to the fault plane and then rotate to higher angles with wall rock deformation. Many naturally occuring examples are found with angles between 30° and 90°. They are referred to as "comb fractures" by Hancock and Barka (1987).



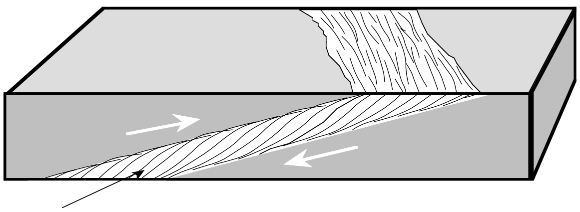
Crescent Marks (bottom) Commonly concave in the direction of movement of the upper (missing) block. They virtually always occur in sets and are usually oriented at a high angle to the fault surface. They are equivalent the "crescentic fractures" formed at the base of glaciers.

diagrams modified after Petit (1987)

[sense of shear is top (missing) block to the right in all the diagrams on this page]

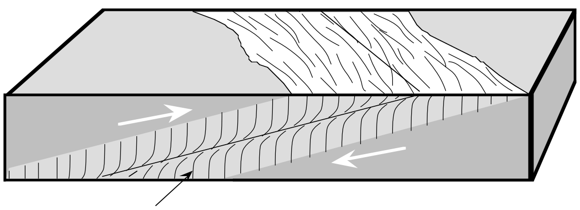
"S-C" Fabrics

Although commonly associated with ductile shear zones, features kinematically identical to S-C fabrics also occur in brittle fault zones. There are two types: (1) those that form in clayey gouge in clastic rocks and (2) those that form in carbonates. They have not been described extensively in the literature. This is somewhat odd because I have found them one of the most useful, reliable, and prevalent indicators.



gouge

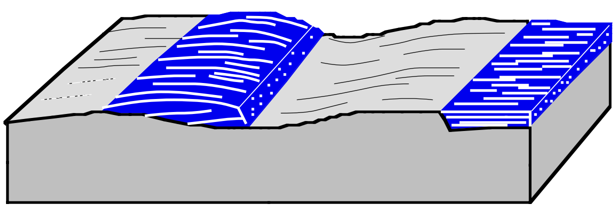
Clayey Gouge fabric (top): Documented by Chester and Logan (1987) and mentioned by Petit (1987). Fabric in the gouge has a sigmoidal shape very similar to S-surfaces in type-1 mylonites. This implies that the maximum strain in the gouge and displacement in the shear zone is along the walls. Abberations along faults may commonly be related to local steps in the walls.



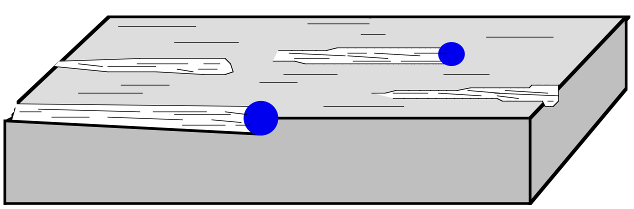
pressure solution cleavage

Carbonate fabric (top): This feature is particularly common in limestones. A pressure solution cleavage is localized in the walls of a fault zone. Because maximum strain and displacement is in the center of the zone rather than the edges, the curvature has a different aspect than the clayey gouge case. The fault surface, itself, commonly has slip-parallel calcite fibers.

Mineral Fibers & Tool Marks



Mineral Fibers and Steps (top): When faulting occurs with fluids present along an undulatory fault surface or one with discrete steps, fiberous minerals grow from the lee side of the asperities where stress is lower and/or gaps open up. These are very common in carbonate rocks and less so in siliceous clastic rocks.



Tool Marks (bottom): This feature is most common in rocks which have clasts much harder that the matrix. During faulting, these clasts gouge the surface ("asperity ploughing" of Means [1987]), producinig trough shaped grooves. Although some attempt to interpret the grooves alone, to make a reliable interpretation, one must see the clast which produced the groove as well. Other- wise, it is impossible to tell if the deepest part of the groove is where the clast ended up or where it was plucked from.

[sense of shear is top (missing) block to the right in all the diagrams on this page]